GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION

2024

10611

MATHEMATICS

(PAPER 1)

MATHEMATICS: Paper 1

10611E

TIME: 3 hours

MARKS: 150

11 pages + 1 information sheet

X05

P.T.O.

INSTRUCTIONS AND INFORMATION

1. This question paper consists of 12 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
7. An information sheet with formulae is included at the end of the question paper.
8. Number the answers according to the numbering system used in this question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 Given: $2k = (x-5)(x-k)$, determine:

1.1.1 k if $x = 2$ (2)

1.1.2 x if $k = 2$ (4)

1.2 Solve for x:

1.2.1 $2x^2 + 3 = 8x$ (correct to TWO decimal places) (4)

1.2.2 $\sqrt{2(x+10)} - 10 = x - 12$ (4)

1.2.3 $3^x(x-5) < 0$ (2)

1.3 Solve the following equations simultaneously:

$\sqrt{3^x}.9^y = 27$ and $x + 4y^2 = 6$ (6)

1.4 The solutions of a quadratic equation are given by

$$x = \frac{-2 \pm \sqrt{2p+5}}{7}.$$

State the value(s) of p for which this equation will have:

1.4.1 Two equal solutions (1)

1.4.2 No real solutions (1)

[24]

QUESTION 2

2.1 Given the quadratic sequence: 0 ; 5 ; 14 ; … ; 779 ; 860

2.1.1 Write down the value of the 4th term, T_4, of this sequence. (1)

2.1.2 Determine an expression for the n^{th} term of this sequence. (4)

2.1.3 Calculate the number of terms in the sequence. (3)

2.2 Determine the sum of the whole numbers between 100 and 1 000 which are divisible by 11. (5)

[13]

QUESTION 3

3.1 Given the geometric sequence: $8(x-2)^2$; $4(x-2)^3$; $2(x-2)^4$; … $x \neq 2$

3.1.1 Determine the value(s) of x where the sequence converges. (3)

3.1.2 Determine the sum to infinity of the series if $x = 2{,}5$. (4)

3.2 Given: $\sum_{k=3}^{12} 3(-2)^{k-2}$

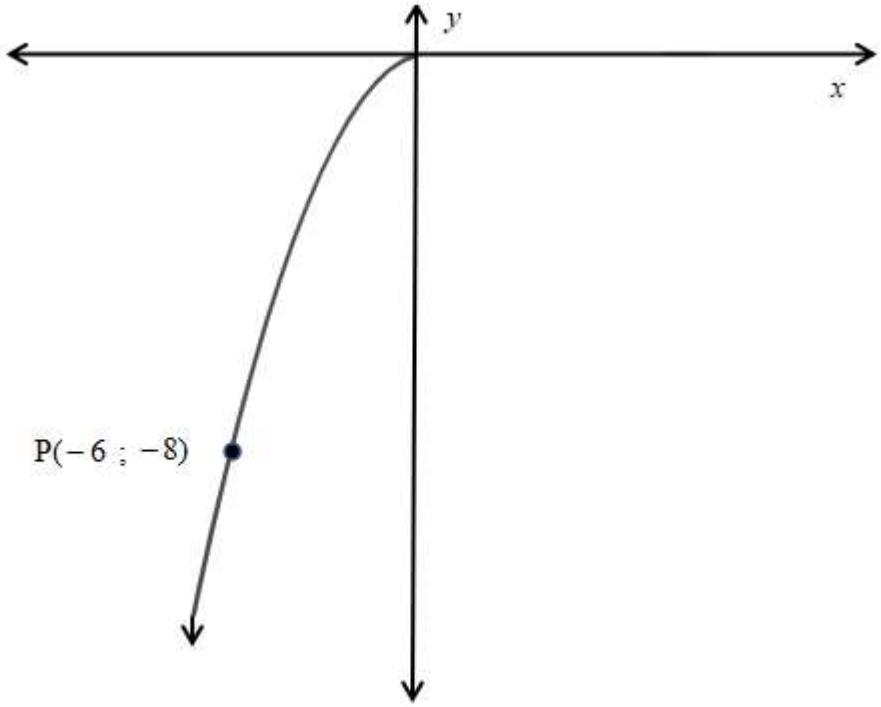
3.2.1 How many terms are there in this series? (1)

3.2.2 Calculate the sum of the series. (3)

[11]

QUESTION 4

The graph of $f(x) = ax^2,\ x \leq 0,$ is sketched below. The point $P(-6\ ;\ -8)$ lies on the graph of f.



4.1 Calculate the value of a. (2)

4.2 Determine the equation of f^{-1}, in the form $y = \ ...$ (2)

4.3 Write down the range of f^{-1}. (1)

4.4 Sketch the graph of f^{-1}. Indicate the coordinates of any point on the graph different to $(0\ ;\ 0)$. (2)

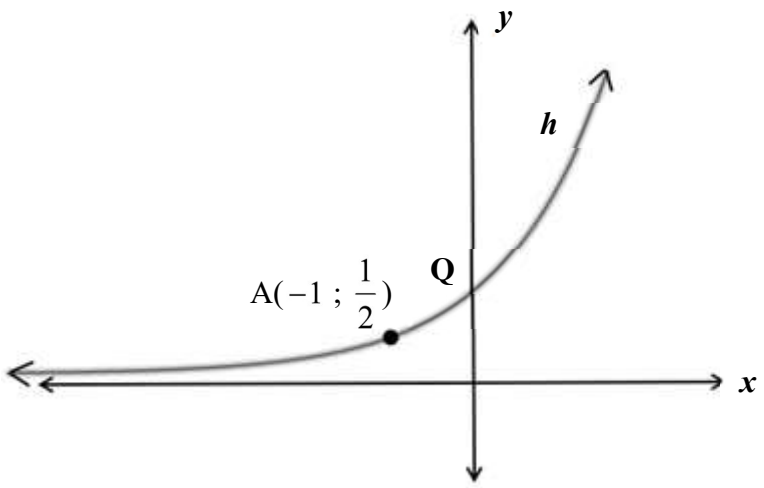
4.5 The graph of f is reflected across the line $y = x$, and thereafter it is reflected across the x-axis.

Determine the equation of the new function in the form $y = ...$ (2)
[9]

QUESTION 5

5.1 The point P(2 ; $\sqrt{3}$) lies in the Cartesian plane. Determine the coordinates of the image of point P if P is rotated about the origin through 90^0 in an anti-clockwise direction. (2)

5.2 The graph of $h(x) = a^x$ is sketched below. A$\left(-1\ ;\ \frac{1}{2}\right)$ is a point on the graph of h.



5.2.1 Substantiate why the coordinates of Q, the y-intercept of h, are (0 ; 1). (2)

5.2.2 Calculate the value of a. (2)

5.2.3 Write down the equation of the inverse function, h^{-1} in the form $y = \ldots$ (2)

5.2.4 Draw a sketch graph of h^{-1}. Indicate the coordinates of TWO points that lie on this graph. (3)

5.2.5 Read off from your graph the values of x for which $\log_2 x > -1$. (2)

5.2.6 If $g(x) = (100).3^x$, determine the values of x for which $h(x) = g(x)$. (3)

5.3 The price (p), in Rands per unit, of EACH item in a consignment of q items, is given by $p = \log\left(10 + \frac{q}{2}\right)$.

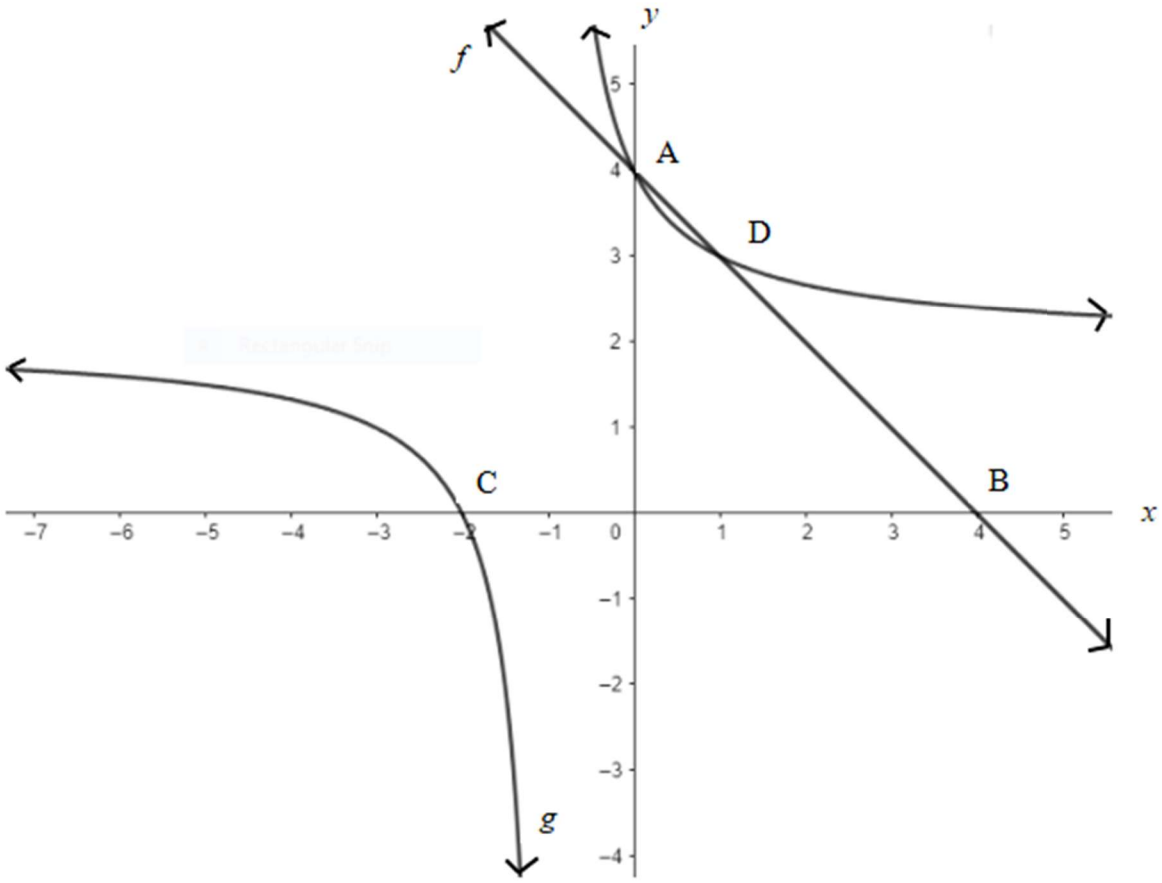
5.3.1 Calculate the value of p and the total price of the consignment when the consignment has 1 980 items. (3)

5.3.2 Determine the number of items in the consignment when the price of each item is R2. (2)

[21]

QUESTION 6

Sketched below are the graphs of $f(x) = -x + 4$ and $g(x) = \frac{2}{x+1} + 2$.



6.1 Write down the domain of g. (1)

6.2 Write down the equations of the asymptotes of g. (2)

6.3 Calculate the coordinates of point D, a point of intersection of g and f. (5)
[8]

QUESTION 7

7.1 At what annual percentage interest rate, compounded quarterly, should a lump sum be invested in order for it to double in 6 years? (3)

7.2 Micaela buys furniture to the value of R10 000. She borrows the money on 1 February 2023 from a financial institution that charges interest at a rate of 9,5% *p.a.* compounded monthly. Micaela agrees to pay monthly instalments of R450. The loan agreement allows Micaela to start paying equal monthly instalments from 01 August 2023.

7.2.1 Calculate the total amount owing to the financial institution on 1 July 2023. (3)

7.2.2 How many months will it take Micaela to pay back the loan? (4)

7.2.3 What is the balance of the loan immediately after Micaela has made the 25th payment? (3)

[13]

QUESTION 8

8.1 If $f(x) = -2x^2 + 3x$, determine $f'(x)$ from first principles. (4)

8.2 Given: $f(x) = \frac{3x^2}{2} - 24\sqrt{x}$. Calculate $f'(9)$. (5)

8.3 A function $g(x) = ax^2 + \frac{b}{x}$ has a minimum value at $x = 4$. The function value at $x = 4$ is 96.

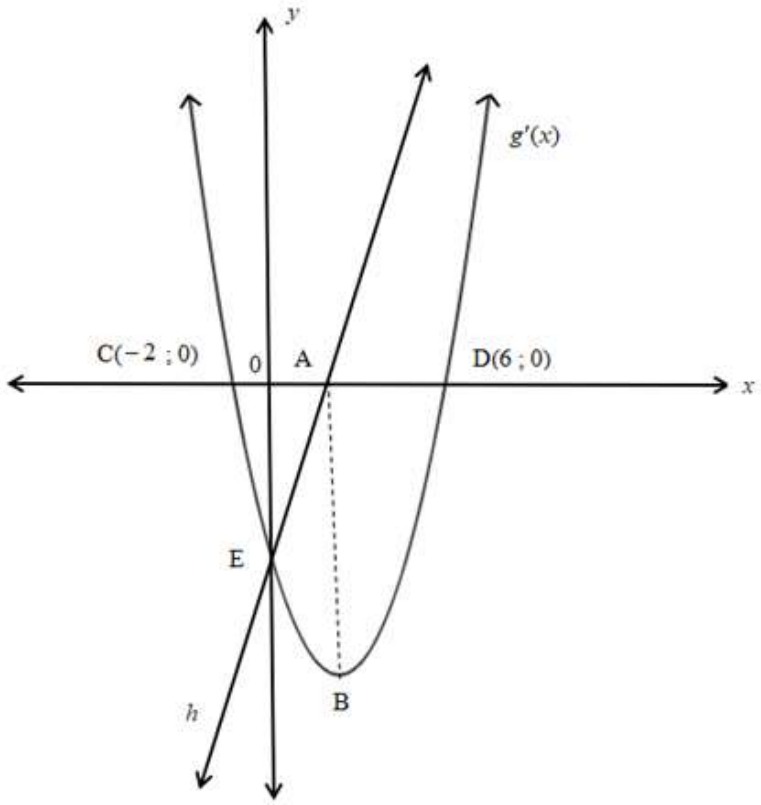
Calculate the values of a and b. (6)

[15]

QUESTION 9

9.1 The graphs of $g'(x) = ax^2 + bx + c$ and $h(x) = 2x - 4$ are sketched below.
The graph of $g'(x) = ax^2 + bx + c$ is the derivative graph of a cubic function g.

- The graphs of h and g' have a common y-intercept at point E.
- C$(-2\ ;\ 0)$ and D$(6\ ;\ 0)$ are the x-intercepts of the graph of g'.
- Point A is the x-intercept of h and point B is the turning point of g'.
- Line AB is parallel to the y-axis.



9.1.1 Write down the coordinates of point E. (1)

9.1.2 Determine the equation of the graph of g' in the form $y = ax^2 + bx + c$. (4)

9.1.3 Write down the x-coordinates of the turning point of g. (2)

9.1.4 Write down the x-coordinate of the point of inflection of the graph of g. (1)

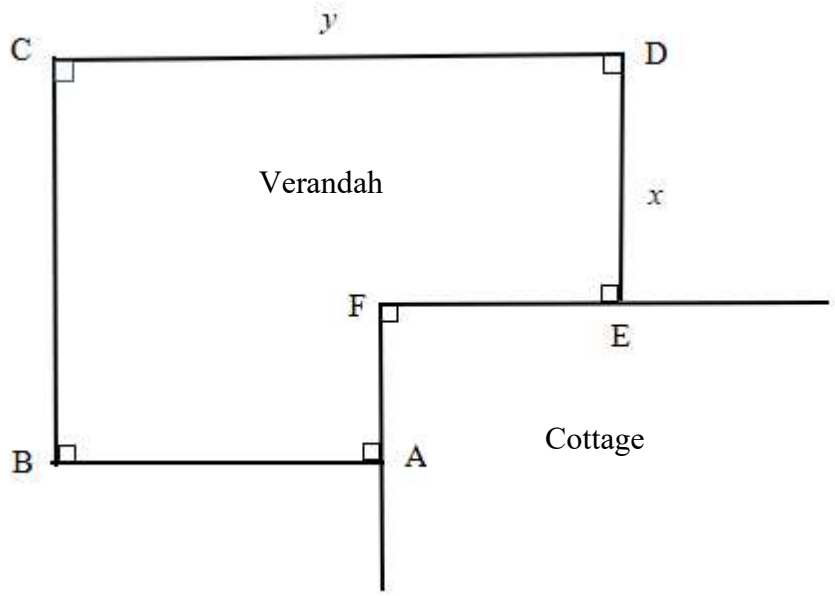
9.1.5 Explain why g has a local maximum at $x = -2$. (2)

9.2 Given: $h(x) = 4x^3 + 5x$

Substantiate whether it is possible to draw a tangent to the graph of h that has a negative gradient. (2)
[12]

QUESTION 10

The diagram below shows the plan for a verandah which is to be built onto the corner of a cottage. A railing ABCDE is to be constructed around the four edges of the verandah.
It is given that AB = DE = x and BC = CD = y, and the length of the railing must be 30 metres.
Calculate the value of x and y for which the veranda will have a maximum area.



[8]

QUESTION 11

Let A and B be two events in a sample space.

Suppose that $P(A) = 0{,}4$; $P(A \text{ or } B) = 0{,}7$ and $P(B) = k$.

11.1 For what value of k are A and B mutually exclusive? (2)

11.2 For what value of k are A and B independent? (3)

[5]

QUESTION 12

12.1 The data obtained from a city's police department indicates that of all motor vehicles reported stolen, 80% were stolen by syndicates to be sold off, and 20% were stolen by individuals for personal use.

Of the vehicles presumed stolen by syndicates:
- 24% were recovered within 48 hours
- 16% were recovered after 48 hours
- 60% were never recovered

Of those vehicles presumed stolen by individuals:
- 38% were recovered within 48 hours
- 58% were recovered after 48 hours
- 4% were never recovered

12.1.1 Draw a tree diagram for the given information above. (3)

12.1.2 Calculate the probability that if a vehicle was stolen in this city, it would be stolen by a syndicate and recovered within 48 hours. (2)

12.1.3 Calculate the probability that a vehicle stolen in this city will not be recovered. (3)

12.2 You have to choose a password for your new "Facebook" profile. The password must be in the format: $\psi\psi\psi@@$ where ψ is any digit (0's are not allowed) and @ is any vowel (a ; e ; i ; o ; u). You may repeat any digit, but you may not repeat a vowel.

How many passwords can be formed? (3)
[11]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r - 1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}\ ;\ -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}\ ;\frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area } \Delta\text{ABC} = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad \mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$