Basic Education

KwaZulu-Natal Department of Basic Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P2

PREPARATORY EXAMINATION

SEPTEMBER 2015

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 150

TIME: 3 hours

**This question paper consists of 12 pages,
1 information sheet and an answer book of 19 pages.**

Copyright Reserved

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.

Copyright Reserved

education
Department:
Education
PROVINCE OF KWAZULU-NATAL

ENQUIRIES: MR D.A. SEWLALL DATE: 29 July 2015

NATIONAL SENIOR CERTIFICATE: PREPARATORY EXAMINATION SEPTEMBER 2015 GRADE 12

TO: THE CHIEF INVIGILATOR OF ALL SCHOOLS OFFERING MATHEMATICS P2:

ERRATA

Please take note of the following change:

Page 5: Question 3: Answer book

Please use the diagram from the question paper to answer the questions in Question 3.

Kindly ensure that candidates are informed of the Errata.

MR R.C. PENNISTON
SENIOR MANAGER
PROVINCIAL EXAMINATIONS SERVICES

08/09/2015
DATE

KWAZULU-NATAL DEPARTMENT OF EDUCATION
Postal Address: Private Bag X01 • EAST END • 4018 • Republic of South Africa
Physical Address: 72 Stalwart Simelane Street • Malgate Building • Durban • 4000
Tel.: +27 31 327 0462 • Web:www.kzneducation.gov.za

QUESTION ONE

Ten athletes took part in a javelin throwing competition. Their height, in *cm*, and their arm span, in *cm*, is shown in the table below.

Athlete	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Height (in *cm*)	156	173	181	174	167	170	169	174	177	168
Arm span (in *cm*)	164	181	193	178	172	178	165	183	190	173

1.1 Represent the height and arm span for each athlete on the scatter plot provided in the answer book. (3)

1.2 Determine the equation of the least squares regression line. (4)

1.3 Use the equation in 1.2 to estimate the arm span of an athlete whose height is 176 *cm*. (2)

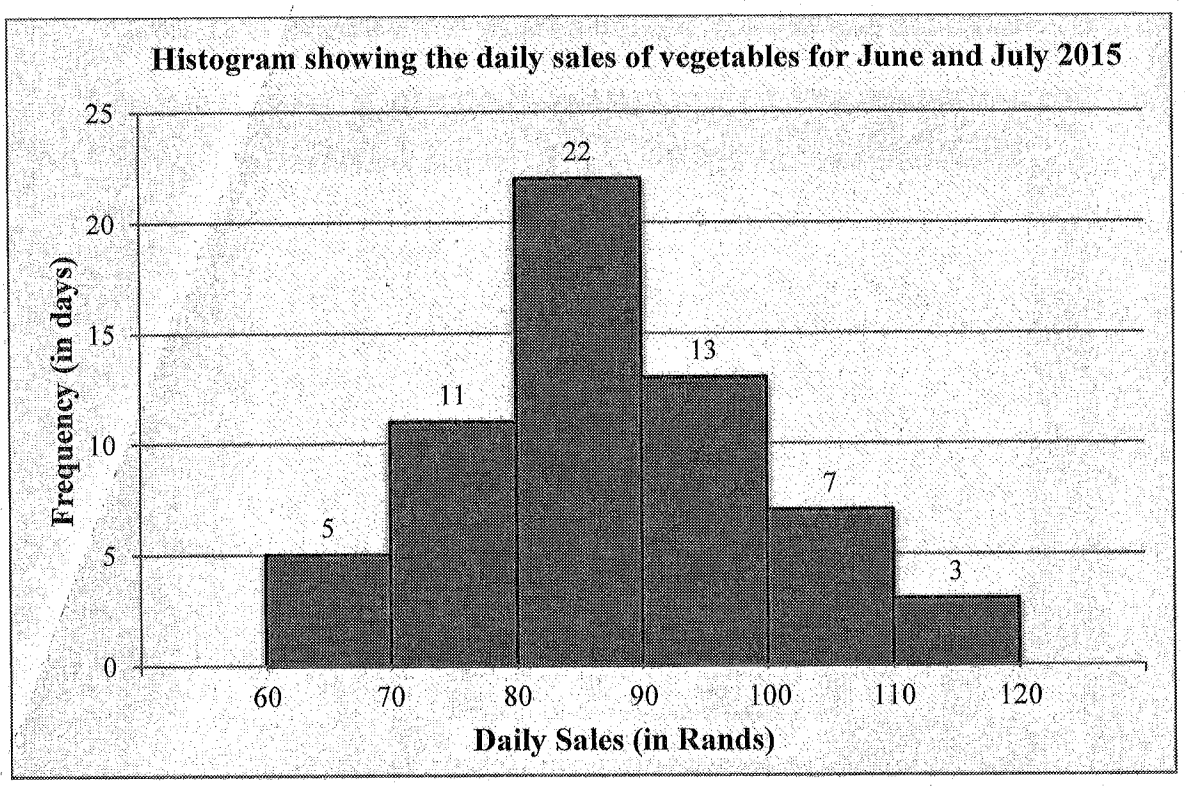
1.4 The correlation coefficient for this set of data is 0,89. Comment on the strength of the relationship between height and arm span. (1)

[10]

Copyright Reserved

QUESTION TWO

Mrs. Fuyane, a street vendor, has kept record of the daily sales of vegetables for the months of June and July 2015. The information is represented in the histogram below.



2.1 Use the information provided in the histogram to complete the frequency and cumulative frequency columns in the table provided in the answer book. (3)

2.2 Use the grid provided in the answer book to draw an ogive (cumulative frequency graph) for the daily sales over this period. (4)

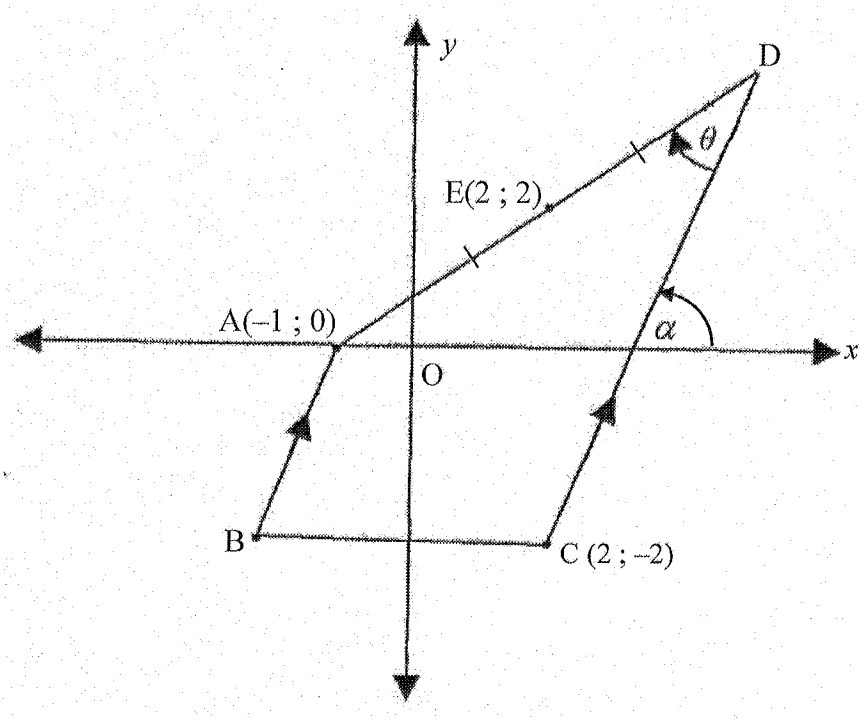
2.3 Use your cumulative frequency graph to estimate the median value for the daily sales. (2)

2.4 Estimate the interval of the upper 25% of the daily sales. (1)
[10]

Copyright Reserved

QUESTION THREE

In the diagram below, A(–1 ; 0), B, C(2 ; –2) and D are the vertices of a trapezium having AB || DC. The length of DC is three times the length of AB (i.e. DC = 3AB). $A\hat{D}C = \theta$. E(2 ; 2) is the midpoint of AD. The angle of inclination of DC is α.



3.1 Determine the coordinates of D. (2)

3.2 Calculate the size of α, correct to ONE decimal place. (3)

3.3 Determine the equation of AB in the form $y = mx + c$. (3)

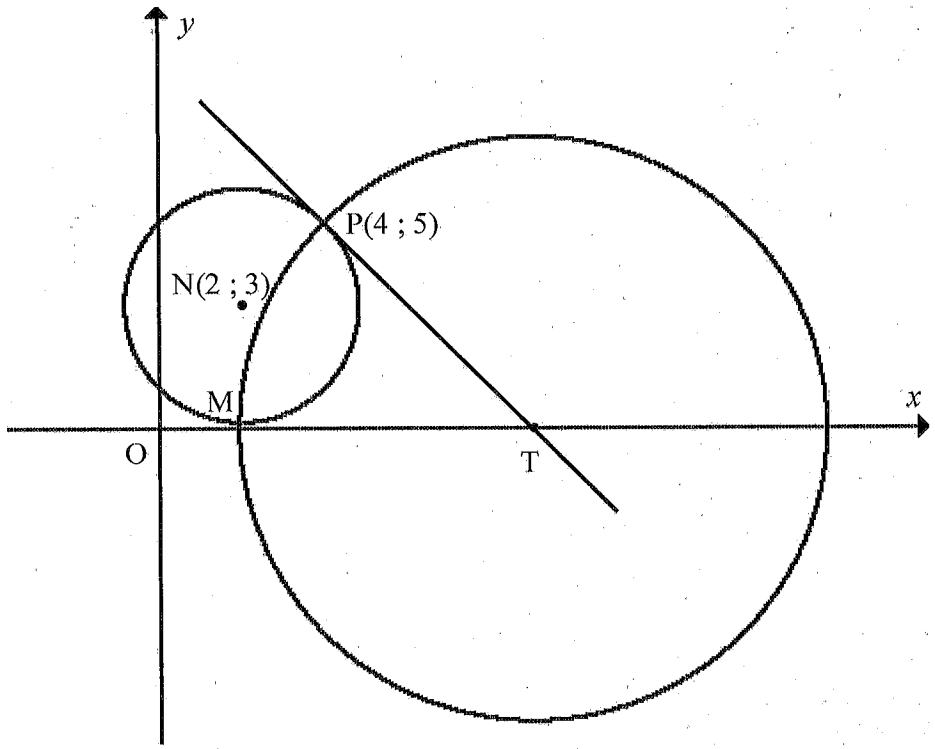
3.4 Calculate the size of θ, correct to ONE decimal place. (3)

3.5 Calculate the coordinates of B. (5)
[16]

Copyright Reserved

QUESTION FOUR

In the figure below, T is a point on the x-axis. A circle having T as its centre intersects a circle having N(2 ; 3) as its centre at P(4 ; 5) and M. TP is a tangent to the circle centre N at P.



4.1 Determine the equation of circle centre N in the form: $(x-a)^2+(y-b)^2=r^2$. (4)

4.2 Calculate the coordinates of T, the x-intercept of PT. (6)

4.3 Calculate the length of PT. Leave your answer in surd form. (2)

4.4 Calculate the area of circle centre T. Give your answer rounded off to the nearest integer. (2)

4.5 Calculate the size of $\hat{NTP}$, correct to ONE decimal place. (2)

4.6 Prove that MNPT is a kite. (3)

4.7 Calculate the size of $\hat{MNP}$, correct to ONE decimal place. (4)
[23]

Copyright Reserved

QUESTION FIVE

5.1 Simplify to a single trigonometric ratio of A:

$$\frac{\tan(180^\circ + A) \cdot \cos(180^\circ - A) \cdot \sin(360^\circ - A)}{\cos(90^\circ - A)} \quad (6)$$

5.2 If $\cos 26 = r$, determine the following in terms of r, in its simplest form:

5.2.1 $\cos 52^\circ$ (3)

5.2.2 $\tan 71^\circ$ (6)

5.3 Prove the identity: $\frac{\sin 2x}{\cos 2x + \sin^2 x} = 2\tan x$ (4)

[19]

QUESTION SIX

6.1 Determine the general solution of: $\cos 2x = \sin(x - 30^\circ)$. (7)

6.2 On the set of axes provided in the answer book, draw the sketch graphs of $f(x) = \cos 2x$ and $g(x) = \sin(x - 30^\circ)$ for $x \in [-180^\circ ; 90^\circ]$. Clearly indicate the coordinates of the turning points and the intercepts with the axes. (6)

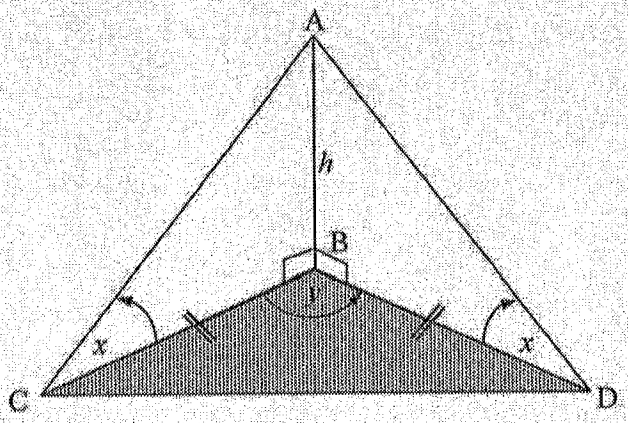
6.3 Write down the values of x for which $g(x) > f(x)$ in the given interval. (3)

[16]

Copyright Reserved

QUESTION SEVEN

In the diagram, C and D represent two ships in the same horizontal plane as B, the base of a lighthouse AB which is h metres high. Each ship is the same distance away from the base of the lighthouse. The angles of elevation from C and D to A are both x. $C\hat{B}D = y$.



7.1 Write BD in terms of h and a trigonometric ratio of x. (2)

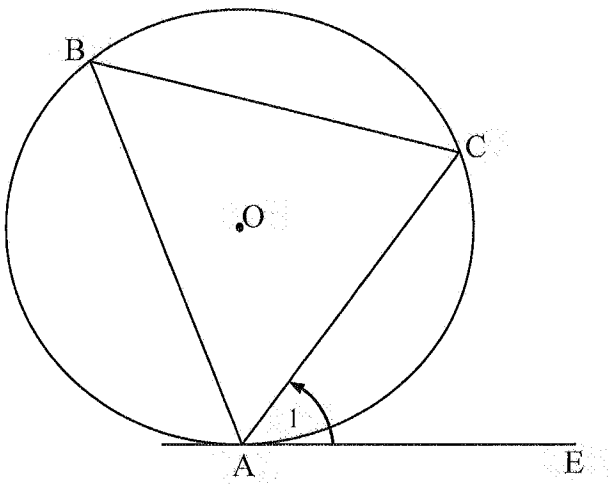
7.2 Hence or otherwise prove $CD^2 = \dfrac{2h^2(1-\cos y)}{\tan^2 x}$ (4)

[6]

NOTE: Give reasons for your statements in these questions 8, 9 and 10.

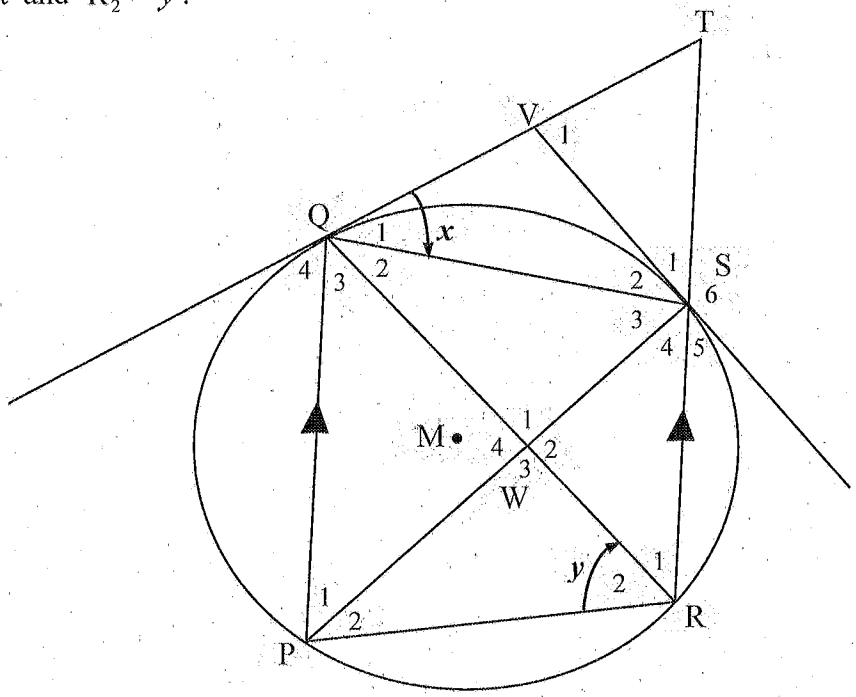
QUESTION EIGHT

8.1 In the diagram below, O is the centre of the circle passing through A, B and C. EA is a tangent to the circle at A. Use this diagram to prove the theorem which states that $E\hat{A}C = A\hat{B}C$. (6)



8.2 In the diagram below, PQ and RS are chords of the circle such that PQ || RS. The tangent to the circle at Q meets RS produced at T and the tangent at S meets QT at V. PS and QR intersect at W. QS and PR are drawn.

Let $\hat{Q}_1 = x$ and $\hat{R}_2 = y$.'



8.2.1 Write down a reason why QV = VS. (1)

8.2.2 Write down the following angles in terms of x:

(a) $\hat{S}_2$ (2)

(b) $\hat{R}_1$ (2)

(c) $\hat{V}_1$ (3)

8.2.3 Show that $\hat{R}_1 = \hat{S}_4$ (4)

8.2.4 Prove that QVSW is a cyclic quadrilateral. (4)

8.2.5 Write down the following angles in terms of y:

(a) $\hat{Q}_4$ (2)

(b) $\hat{T}$ (2)

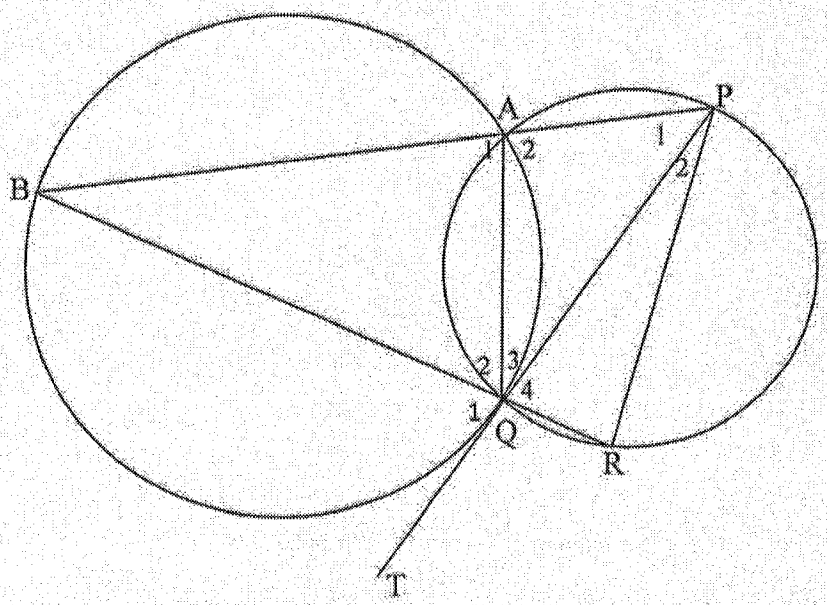
8.2.6 If M is the centre of the circle, hence prove that PMWR is a cyclic quadrilateral. (2)

[28]

Copyright Reserved

QUESTION NINE

In the diagram below, PQT is a tangent to the larger circle ABQ at Q. A smaller circle intersects the larger circle at A and Q. BAP and BQR are straight lines with P and R on the smaller circle. AQ and PR are drawn.



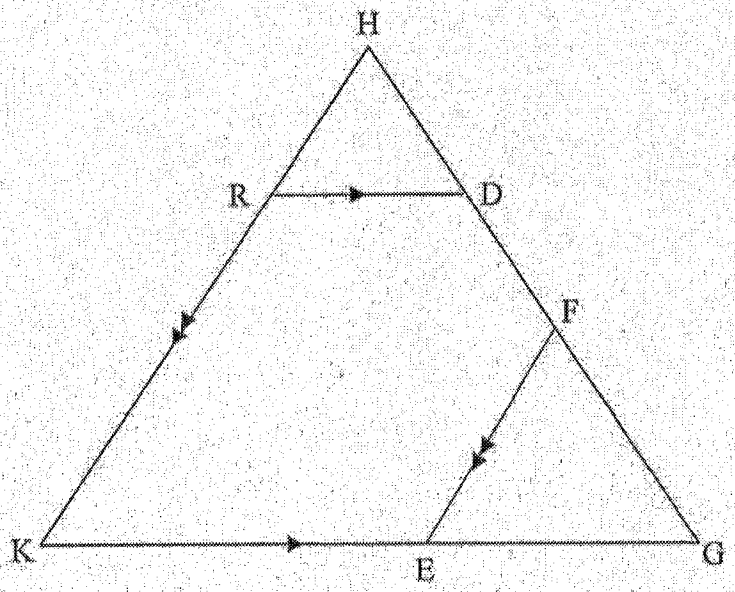
9.1 Prove that PQ = PR (7)

9.2 Prove that Δ PBQ ||| Δ PQA. (4)

9.3 Prove that the lengths of PA, PR and PB (in this order) form a geometric sequence. (3)
[14]

QUESTION TEN

In the diagram below, ΔGHK is drawn having the point R on KH and the points D and F on GH such that RD || KG and EF || KH. It is also given that RH = 3 units, RK = 9 units, HD = 2 units and GE : EK = 1 : 2.



Calculate the lengths of:

10.1 DG (3)

10.2 FD. (5)
[8]

TOTAL: 150

Copyright Reserved

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \; ; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$\mathrm{P}(A) = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A\ or\ B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A\ and\ B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

Copyright Reserved

Basic Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P2

PREPARATORY EXAMINATION

SEPTEMBER 2015

MEMORANDUM

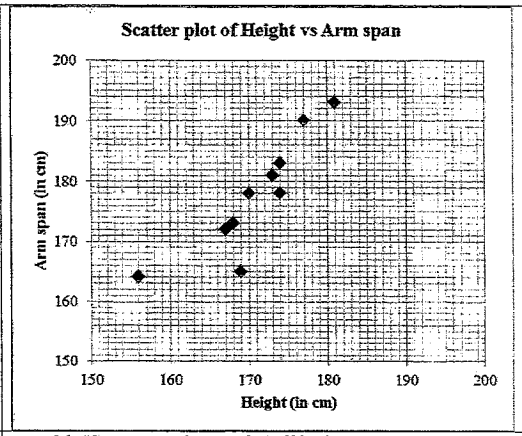
NATIONAL SENIOR CERTIFICATE

GRADE 12

N.B. This memorandum consists of 15 pages including this page.

Copyright Reserved

QUESTION 1

1.1		✓ 1 – 4 points correct ✓ 5 – 9 points correct ✓ all points correct (3)
1.2	$a = -36,58$ ($a = -36,57689....$) $b = 1,25$ ($b = 1,25381....$) $\hat{y} = -36,58 + 1,25x$	✓✓ a ✓ b ✓ equation (4)
1.3	$\hat{y} = -36,58 + 1,25(176)$ $= 183,42$	✓ substitute 176 ✓ answer (2)
1.4	There is strong, positive correlation between height and arm span.	✓ strong, positive (1) **[10]**

Copyright Reserved

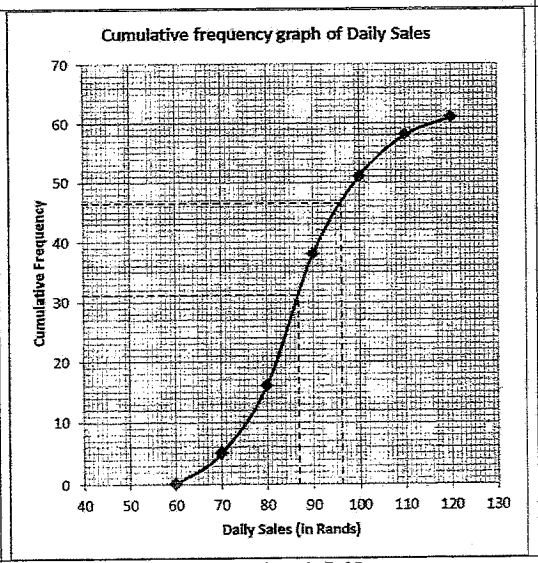
QUESTION 2

2.1

Daily Sales	Frequency	Cumulative Frequency
$60 \le x < 70$	5	5
$70 \le x < 80$	11	16
$80 \le x < 90$	22	38
$90 \le x < 100$	13	51
$100 \le x < 110$	7	58
$110 \le x < 120$	3	61

✓ first two cumulative frequencies correct
✓ next two cumulative frequencies correct
✓ remainder correct (total = 61)
(3)

2.2



✓ grounding at 0
✓ plotting cumulative frequencies at upper limits
✓ points correct
✓ smooth shape of curve
(4)

2.3 The median for the data is approximately R 87 .

✓ reading of from graph
✓ R87
(2)

2.4 The upper 25% interval is R96 to R120
(Range to accept: 94 to 120)

✓ 96 to 120
(1)

[10]

Copyright Reserved

QUESTION 3

3.1

$\frac{x_D - 1}{2} = 2 \qquad \frac{y_D + 0}{2} = 2$

$x_D = 5 \qquad x_D = 4$

D(5 ; 4)

✓ $x_D = 5$
✓ $y_D = 4$
(2)

3.2

$m_{CD} = \frac{4-(-2)}{5-2}$

$= 2$

$\tan \alpha = 2$

$\therefore \alpha = 63{,}4°$

✓ substitution into gradient formula
✓ $\tan\alpha = 2$
✓ answer (3)

3.3

$m_{AB} = m_{CD} = 2$ AB||CD, equal gradients

$y = 2x + c$

$0 = 2(-1) + c$

$c = 2$

$y = 2x + 2$

✓ $m_{AB} = 2$
✓ subst (–1 ; 0)
✓ answer
(3)

3.4

$m_{AD} = \frac{4-(0)}{5-(-1)}$

$= \frac{2}{3}$

$\tan(\angle \text{ of inclination of AD}) = \frac{2}{3}$

$\angle$ of inclination of AD = 33,7°

$\therefore \theta = 63{,}4° - 33{,}7°$

$= 29{,}7°$

✓ $m = \frac{2}{3}$
✓33,7°
✓29,7°
(3)

Copyright Reserved

3.5	$3AB = DC$ $\therefore 9AB^2 = DC^2$ $9\left[(x+1)^2 + (y-0)^2\right] = (5-2)^2 + (4+2)^2$ $9\left[(x+1)^2 + y^2\right] = 45$ $AB \parallel DC, \therefore \frac{y-0}{x+1} = 2$ $(x+1)^2 + y^2 = 5$ .....(1) $y = 2x + 2$ ..... (2) Substitute (2) in (1) $(x + 1)^2 + (2x + 2)^2 = 5$ $x^2 + 2x + 1 + 4x^2 + 8x + 4 - 5 = 0$ $5x^2 + 10x = 0$ $5x(x + 2) = 0$ $x \neq 0$; $x = -2$ Substitute $x = -2$ in (1) $y = -2$ B(–2; –2)	✓ substitution ✓ substitute $y = 2x + 2$ ✓ standard form ✓ $x = -2$ ✓ $y = -2$ (5) **[16]**

QUESTION 4

4.1	$r^2 = (2-4)^2 + (3-5)^2$ $= 8$ $(x-2)^2 + (y-3)^2 = 8$	✓ subst into distance formula ✓ 8 ✓ $(x-2)^2$ ✓ $(y-3)^2$ (4)
4.2	$m_{NP} = \frac{5-3}{4-2} = 1$ $m_{PT} = -1$ NP ⊥ PT, product of gradients = – 1 $y = -x + c$ $5 = -4 + c$ $c = 9$ $y = -x + 9$ $0 = -x + 9$ $x = 9$ $\therefore$ T(9 ; 0)	✓ $m_{NP} = 1$ ✓ $m_{PT} = -1$ ✓ subst (5; 4) ✓ $c = 9$ ✓ $y = -x + 9$ ✓coordinates of T (6)
4.3	$PT = \sqrt{(9-4)^2 + (5-0)^2}$ $= \sqrt{50}$ $= 5\sqrt{2}$	✓ substitution into distance formula ✓ $\sqrt{50}$ or $5\sqrt{2}$ (2)
4.4	Area $= \pi \times PT^2$ $= \pi \times 50$ $= 157$	✓ substitution into area formula ✓ 157 (2)
4.5	$\tan N\hat{P}T = \frac{\sqrt{8}}{\sqrt{50}}$ $N\hat{P}T = 21{,}8°$	✓ $\tan N\hat{P}T = \frac{\sqrt{8}}{\sqrt{50}}$ ✓21,8° (2)
4.6	NP = NM radii PT = TM radii $\therefore$ MNPT is a kite two pairs of adjacent sides equal in length	✓ S/R ✓ S/R ✓ reason (3)

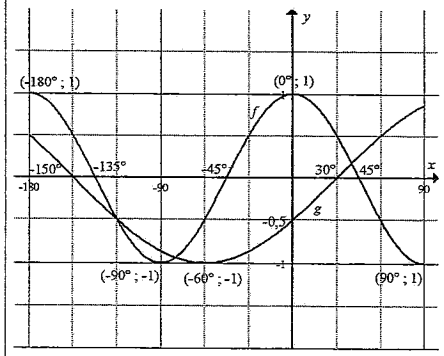
4.7	$N\hat{T}P = N\hat{T}M = 21,8°$ diagonal of kite $N\hat{P}T = N\hat{M}T = 90°$ tangent perpendicular to radius $N\hat{M}P = 360° - 90° - 90° - 43,6°$ angles in quadilateral $= 136,4°$ **OR** $NP \perp PT$ (rad $\perp$ tan) $\therefore T\hat{N}P = 68,2°$ ($\angle$ sum Δ) $T\hat{N}P = T\hat{N}M$ (prop of kite) $\therefore M\hat{N}P = 2(68,2°)$ $\therefore M\hat{N}P = 136,4°$	✓ S/R ✓ S/R ✓ S/R ✓ answer (4) ✓ S/R ✓ S/R ✓ S/R ✓ answer (4) **[23]**

QUESTION 5

5.1	$\frac{\tan(180° + A).\cos(180° - A).\sin(360° - A)}{\cos(90° - A)}$ $= \frac{(\tan A)(-\cos A)(-\sin A)}{\sin A}$ $= \frac{\sin A}{\cos A}.\cos A$ $= \sin A$	✓ tan A, ✓ – cos A, ✓ – sin A, ✓ sin A ✓ $\frac{\sin A}{\cos A}$ ✓ answer (6)
5.2.1	$\cos 52° = \cos 2(26°)$ $= 2\cos^2 26° - 1$ $= 2(r)^2 - 1$ $= 2r^2 - 1$	✓ writing 52° in terms of 26° ✓ expansion ✓ answer (3)
5.2.2	$\tan 71° = \frac{\sin 71°}{\cos 71°}$ $= \frac{\sin(45° + 26°)}{\cos(45° + 26°)}$ $= \frac{\sin 45°\cos 26° + \cos 45°\sin 26°}{\cos 45°\cos 26° - \sin 45°\sin 26°}$ $= \frac{\left(\frac{\sqrt{2}}{2}\right)r + \left(\frac{\sqrt{2}}{2}\right)\left(\sqrt{1-r^2}\right)}{\left(\frac{\sqrt{2}}{2}\right)r - \left(\frac{\sqrt{2}}{2}\right)\left(\sqrt{1-r^2}\right)}$ $= \frac{\left(\frac{\sqrt{2}}{2}\right)\left(r + \sqrt{1-r^2}\right)}{\left(\frac{\sqrt{2}}{2}\right)\left(r - \sqrt{1-r^2}\right)}$ $= \frac{r + \sqrt{1-r^2}}{r - \sqrt{1-r^2}}$	✓ identity ✓ writing in terms of 26° ✓ expansions ✓ $\sqrt{1-r^2}$ ✓ substitution ✓ answer (6)

5.3	$LHS = \dfrac{\sin 2x}{\cos 2x + \sin^2 x}$ $= \dfrac{2\cos x \sin x}{\cos^2 x - \sin^2 x + \sin^2 x}$ $= \dfrac{2\cos x \sin x}{\cos^2 x}$ $= \dfrac{2\sin x}{\cos x}$ $= 2\tan x$ $= RHS$	✓ identity for sin2x ✓ identity for cos 2x ✓ simplification ✓ identity (4)
	OR $LHS = \dfrac{\sin 2x}{\cos 2x + \sin^2 x}$ $= \dfrac{2\cos x \sin x}{2\cos^2 x - 1 + \sin^2 x}$ $= \dfrac{2\cos x \sin x}{2\cos^2 x - (1 - \sin^2 x)}$ $= \dfrac{2\cos x \sin x}{2\cos^2 x - \cos^2 x}$ $= \dfrac{2\cos x \sin x}{\cos^2 x}$ $= \dfrac{2\sin x}{\cos x}$ $= 2\tan x$ $= RHS$	✓ identity for sin2x ✓ identity for cos 2x ✓ simplification ✓ identity (4)
	OR $LHS = \dfrac{\sin 2x}{\cos 2x + \sin^2 x}$ $= \dfrac{2\cos x \sin x}{1 - 2\sin^2 x + \sin^2 x}$ $= \dfrac{2\cos x \sin x}{1 - \sin^2 x}$ $= \dfrac{2\cos x \sin x}{\cos^2 x}$ $= \dfrac{2\sin x}{\cos x}$ $= 2\tan x$ $= RHS$	✓ identity for sin2x ✓ identity for cos 2x ✓ simplification ✓ identity (4) **[19]**

QUESTION 6

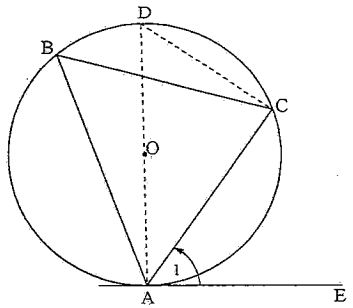
6.1	$\cos 2x = \sin(x - 30°)$ $= \cos[90° - (x - 30°)]$ $= \cos(120° - x)$ key angle $= 120° - x$ $2x = 120° - x + n.360°; n \in Z$ $3x = 120° + n.360°; n \in Z$ $x = 40° + n.120°; n \in Z$ or $2x = 360° - (120° - x) + n.360°; n \in Z$ $2x = 240° + x + n.360°; n \in Z$ $x = 240° + n.360°; n \in Z$	✓ using co-ratio ✓ $120° - x$ ✓ $2x = 120° - x + n.360°$ ✓ $x = 40° + n.120°$ ✓ $-2x = 360° - (120° - x) + n.360°$ ✓ $x = 240° + n.360°$ ✓ $n \in Z$ (7)
6.2		*f* ✓ *x*-intercepts ✓ turning points ✓ shape *g* ✓ intercepts ✓ turning points ✓ shape (6)
6.3	$-120° < x < -80°$ or $40° < x \leq 90°$	✓ critical values: $-120°$; $-80°$ ✓ critical values: $40°$; $90°$ ✓ correct notation (3) **[16]**

QUESTION 7

7.1	$\tan x = \frac{AB}{BD}$ $= \frac{h}{BD}$ $BD = \frac{h}{\tan x}$	✓ using tan ratio ✓ $BD = \frac{h}{\tan x}$ (2)
7.2	BC = BD $CD^2 = BC^2 + BD^2 - 2BC.BD.\cos y$ $= \left(\frac{h}{\tan x}\right)^2 + \left(\frac{h}{\tan x}\right)^2 - 2\left(\frac{h}{\tan x}\right)\left(\frac{h}{\tan x}\right).\cos y$ $= \frac{h^2}{\tan^2 x} + \frac{h^2}{\tan^2 x} - \frac{2h^2}{\tan^2 x}.\cos y$ $= \frac{2h^2}{\tan^2 x} - \frac{2h^2}{\tan^2 x}.\cos y$ $= \frac{2h^2}{\tan^2 x}(1 - \cos y)$ $= \frac{2h^2(1-\cos y)}{\tan^2 x}$	✓ using cosine formula ✓ substitution ✓ simplification ✓ common factor (4) [6]

QUESTION 8

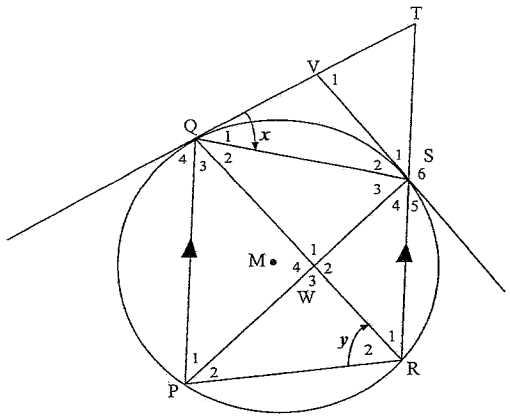
8.1



Construction: Draw diameter AD. Join D to C ✓ construction

Proof:

$E\hat{A}C + D\hat{A}C = 90°$	tan ⊥ rad	✓ S / R
$D\hat{C}A = 90°$	diameter subtends right angle	✓ S / R
$A\hat{D}C + D\hat{A}C = 90°$	sum of angles in triangle	✓ S / R
$\therefore E\hat{A}C = A\hat{D}C$		
But $A\hat{B}C = A\hat{D}C$	angles in same segment	✓ S ✓ R (6)
$\therefore E\hat{A}C = A\hat{B}C$		

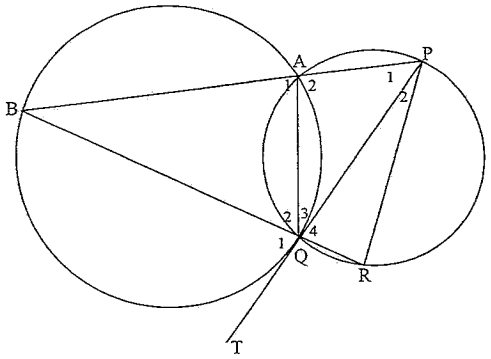


8.2.1	Two tangents drawn from the same external point are equal in length.		✓ answer (1)
8.2.2 (a)	$\hat{S}_2 = x$	angles opposite equal sides	✓ S ✓ R (2)
8.2.2 (b)	$\hat{R}_1 = x$	tan-chord theorem	✓ S ✓ R (2)
8.2.2 (c)	$\hat{V}_1 = \hat{S}_2 + \hat{R}_1$ $= x + x$ $= 2x$	exterior angle of triangle	✓ S ✓ R ✓ answer (3)
8.2.3	$\hat{R}_1 = \hat{Q}_3 = x$ $\hat{Q}_3 = \hat{S}_4 = x$ $\therefore \hat{R}_1 = \hat{S}_4$	alt angles PQ ‖ RS angles in same segment	✓ S ✓ R ✓ S ✓ R (4)
8.2.4	$\hat{W}_1 = \hat{S}_4 + \hat{R}_1$ $= 2x$ $\hat{W}_1 = \hat{V}_1$ $\therefore$ QVSW is a cyclic quad	exterior angle of triangle converse: exterior angle of cyclic quad	✓ S ✓ R ✓ S ✓ R (4)
8.2.5 (a)	$\hat{Q}_4 = y$	tan-chord theorem	✓ S ✓ R (2)
8.2.5 (b)	$\hat{T} = \hat{Q}_4 = y$	corresponding angles PQ ‖ RS	✓ S ✓ R (2)
8.2.6	$\hat{Q}_3 = x$ $\hat{M} = 2x$ $\hat{M} = \hat{W}_3 = 2x$ $\therefore$ PMWR is a cyclic quad	proven in 8.2.3 angle at centre = twice angle at circumference converse: angles in same segment	 ✓ S ✓ R (2)

Copyright Reserved

		[28]

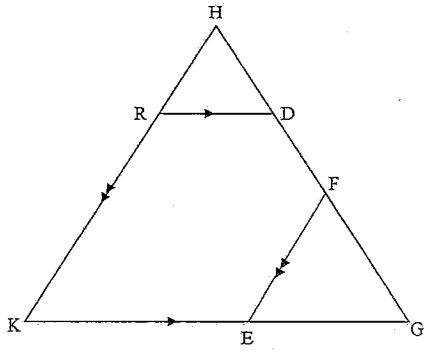
QUESTION 9



9.1	$\hat{Q}_4 = \hat{Q}_1$	vert opp angles	✓ S ✓ R
	$\hat{Q}_1 = \hat{A}_1$	tan-chord theorem	✓ S ✓ R
	$\hat{A}_1 = \hat{R}$	ext angle of cyclic quad	✓ S ✓ R
	$\Rightarrow \hat{Q}_4 = \hat{R}$		
	$\therefore$ PQ = PR	sides opp equal angles	✓ R (7)
9.2	In ΔPBQ and ΔPQA		
	(i)	$\hat{P}_1$ is common	✓ S
	(ii)	$\hat{B} = \hat{Q}_2$ tan-chord theorem	✓ S ✓ R
	(iii)	$P\hat{Q}B = \hat{A}_2$ remaining angles in triangle	✓ S / R
	$\therefore \Delta PBQ \,|||\, \Delta PQA$	equiangular (4)	
9.3	$\frac{PA}{PQ} = \frac{PQ}{PB}$	from 9.2	✓ deduction
	But PQ = PR		✓ PQ = PR
	$\therefore \frac{PA}{PR} = \frac{PR}{PB}$		✓ conclusion in full
	$\therefore$ PA, PR and PB form a geometric sequence	the ratio is constant (3)	
			[14]

Copyright Reserved

QUESTION 10



10.1	In Δ HKG: $\frac{DG}{HD} = \frac{RK}{RH}$ (RD ‖ KG) $\frac{DG}{2} = \frac{9}{3}$ $\therefore DG = 6$ units	✓ S / R ✓ substitution ✓ answer (3)
10.2	Let FD = y $\therefore FG = 6 - y$ $\frac{GF}{FH} = \frac{GE}{EK}$ (DE ‖ HK) $\frac{6-y}{y+2} = \frac{1}{2}$ $2(6-y) = y + 2$ $12 - 2y = y + 2$ $-3y = -10$ $\therefore y = \frac{10}{3}$ = FD	✓ statement ✓ S / R ✓ substitution ✓ simplification ✓ answer (5) **[8]**

TOTAL: 150

Copyright Reserved