GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION/*JUNIE EKSAMEN*
GRADE/*GRAAD* 12

2024

MARKING GUIDELINES/*NASIENRIGLYNE*

TECHNICAL MATHEMATICS/*TEGNIESE WISKUNDE*

(PAPER/*VRAESTEL* 1)

Marking Codes/*Nasienkodes*	
A	Accuracy/*Akkuraatheid*
CA	Consistent accuracy/*Volgehoue Akkuraatheid*
M	Method/*Metode*
R	Rounding/*Afronding*
NPR	No penalty for rounding/*Geen penalisering vir afronding*
NPU	No penalty for units omitted/*Geen penalisering vir weggelate eenhede*
S	Simplification/*Vereenvoudiging*
SF	Substitution in correct formula/*Vervanging in korrekte formule*
ST	Statement/*Stelling*
ST/RE	Statement and Reason/*Stelling en Rede*
RE	Reason/*Rede*

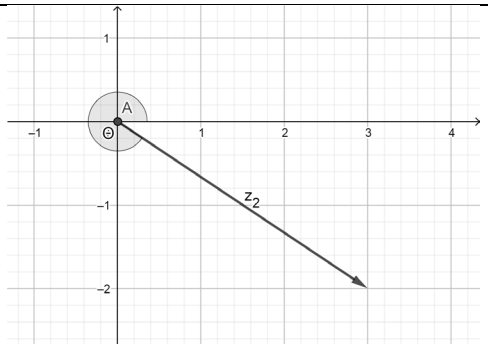
NOTE/LET WEL:

- If a candidate answers a question TWICE, mark only the FIRST attempt/
  Indien 'n kandidaat 'n vraag TWEE keer beantwoord, sien slegs die eerste poging na.
- If a candidate has crossed out an attempt to answer the question and did not redo it, mark the crossed-out version/
  Indien 'n kandidaat 'n antwoord doodgetrek het, maar dit nie oorgedoen het nie, merk die doodgetrekte weergawe.
- Consistent accuracy applies in all aspects of the marking guidelines, where indicated.
  Volgehoue akkuraatheid is deurgaans op alle aspekte van die nasienriglyne van toepassing soos aangedui..

QUESTION/*VRAAG* 1					CL
1.1	1.1.1	$x(x-10)=0$ $x=0$ or/*of* $x=10$	 ✓ $x=0$ A ✓ $x=10$ A	(2)	1D
	1.1.2	$7x^2-5x-6=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-(-5)\pm\sqrt{(-5)^2-4(7)(-6)}}{2(7)}$ $x=\frac{5\pm\sqrt{193}}{14}$ $x=1{,}35$ or/*of* $x=-0{,}64$	 ✓ SF A ✓ $x=1{,}35$ CA ✓ $x=-0{,}64$ CA	(3)	1D
	1.1.3	$(x+2)(2x-4)\geq 0$ $x=-2$ or/*of* $x=2$ $x\leq -2$ or/of $x\geq 2$	 ✓ $x=-2$ (critical value) A or/*of* $x=2$ (critical value) ✓ Notation/*Notasie* CA ✓ both end points/ *beide eindpunte* CA (Graphical representation $\frac{3}{3}$	(3)	2D
1.2	$x-2y=1$ and/*en* $x^2+y^2=9+2xy$ $x=1+2y$ $(1+2y)^2+y^2=9+2y(1+2y)$ $1+4y+4y^2+y^2=9+2y+4y^2$ $1+4y+4y^2+y^2-9-2y-4y^2=0$ $y^2+2y-8=0$ $(y+4)(y-2)=0$	 ✓ $x=1+2y$ A ✓ SF CA ✓ Standard form/ *Standaardvorm* CA ✓ Factors/*Faktore*		4D

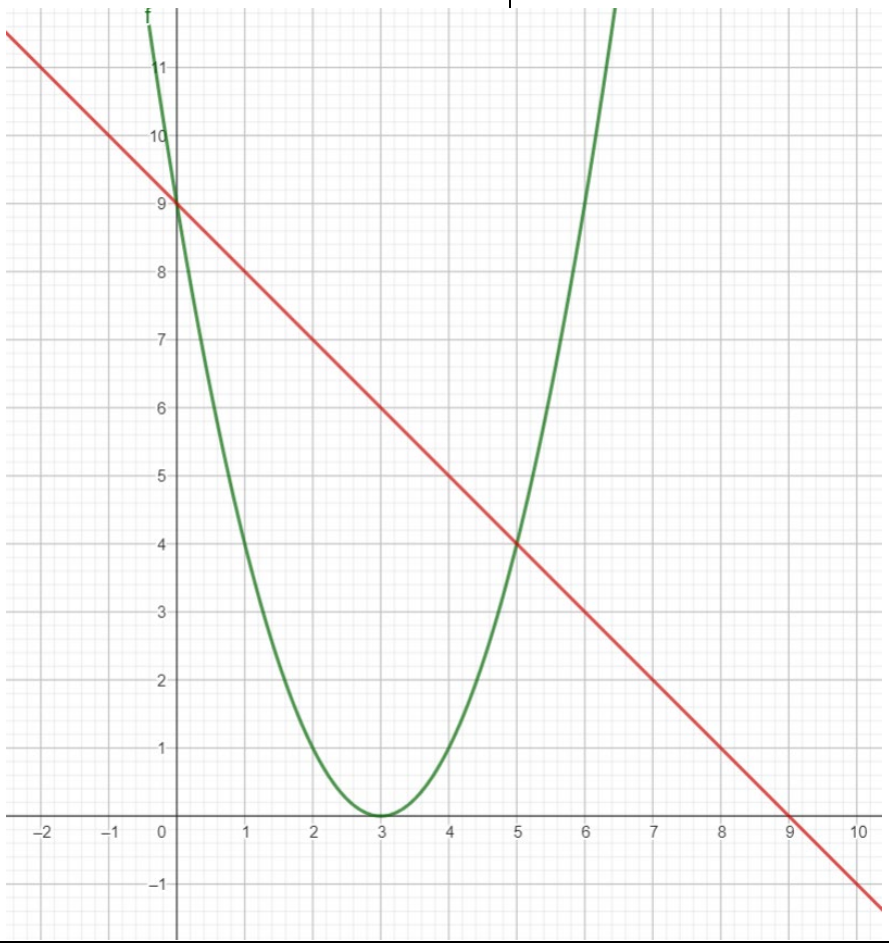
	OR/*OF*		**OR/*OF***			
	$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$					
	$y = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-8)}}{2(1)}$		✓ SF	A		
	$y = \frac{-2 \pm \sqrt{36}}{2}$					
	$y = 2 \quad of \quad y = -4$		✓ Both y-values *Beide y-waardes*	CA		
	$x = 1 + 2(2)$ OR/*OF* $x = 1 + 2(-4)$					
	$x = 5$ OF $x = -7$		✓ Both x-values *Beide x-waardes*	CA	(6)	
1.3	1.3.1	$v = \sqrt{2gh}$ $v^2 = 2gh$ $\frac{v^2}{2g} = h$	✓ v^2 ✓ h the subject/ *h die onderwerp*	M CA	(2)	1D
	1.3.2	$\frac{v^2}{2g} = h$ $\frac{(20)^2}{2(9{,}80)} = h$ $h = 20\ m$ $B = 20\ m$ $A = 30\ m$	✓ Substitution/*Substitusie* ✓ $B = 20\ m$ ✓ $A = 30\ m\ or\ 10\ m$ **If learners swopped the values of A and B, full marks./*Indien die leerders die waardes van A en B omgeruil het, volpunte.***	CA CA CA	(3)	1D
1.4	1.4.1	$1\ 000\ 000_2$	✓$1\ 000\ 000_2$	A	(1)	1D
	1.4.2	64	✓ Answer/*Antwoord*	CA	(1)	1E
					[21]	

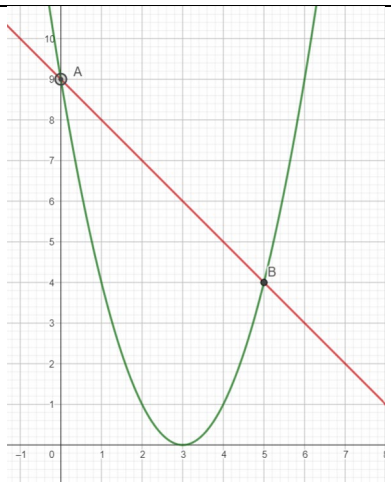
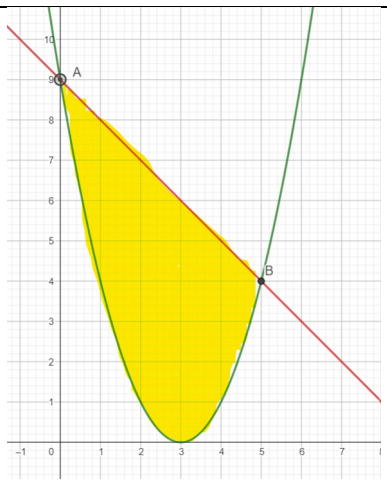
QUESTION/*VRAAG* 2					
2.1	2.1.1	$\Delta = b^2 - 4ac$ $= (3)^2 - 4(1)(0)$ $= 9$	✓ SF A ✓ $\Delta = 9$ A	(2)	2D
	2.1.2	Roots are real, unequal, and rational. *Wortels is reël, ongelyk en rasionaal.*	✓ Answer/*Antwoord* CA If only two of the three descriptions- full marks	(1)	2D
2.2	$mx^2 - 12x + 9 = 0$ $\Delta = b^2 - 4ac$ $= (-12)^2 - 4(m)(9)$ $= 144 - 36\,m$ $\therefore \Delta = 144 - 36\,m$ **For equal roots/*Vir* gelyke *wortels*: $\Delta = 0$** $144 - 36\,m = 0$ $m = 4$		✓ SF A ✓ $\Delta = 144 - 36m$ CA ✓ $\Delta = 0$ A ✓ $m = 4$ CA	(4)	3E
				[7]	
QUESTION/*VRAAG* 3					
3.1	3.1.1	$-2a^0 \times x^{-3} \div x^7$ $= -2 \times x^{-10}$ $= \frac{-2}{x^{10}}$ **OR/*OF*** $-2a^0 \times x^{-3} \div x^7$ $= -2(1) \times \frac{1}{x^3} \div \frac{x^7}{1}$ $= -2(1) \times \frac{1}{x^3} \times \frac{1}{x^7}$ $= \frac{-2}{x^{10}}$	✓ -2 A ✓ x^{10} (Must be a denominator) A A ✓ -2 A ✓ x^{10}	(2)	1E
	3.1.2	$\sqrt{27x^{10}} \times \sqrt{18x^{-2}}$ $= \sqrt{9 \times 3x^{10}} \times \sqrt{9 \times 2x^{-2}}$ $= 3\sqrt{3}x^5 \times 3\sqrt{2}x^{-1}$ $= 9\sqrt{6}x^4$	✓ $3\sqrt{3}x^5$ ✓ $3\sqrt{2}x^{-1}$ ✓ Answer A **AO $\frac{1}{3}$** CA	(3)	1D

	3.1.3	$\frac{5^{4x}+2.5^{4x}}{625^x}$ $=\frac{5^{4x}+2.5^{4x}}{(5^4)^x}$ $=\frac{5^{4x}(1+2)}{5^{4x}}$ $=3$	✓ Prime factors/ *Priemfaktore* 5^4 M ✓ Factorisation/ *Faktorisering* CA ✓ Answer/*Antwoord* CA **AO $\frac{1}{3}$**		(3)	2E
3.2	$\log_2(1-x)+\log_2(5+x)-3=0$ $\log_2(1-x)+\log_2(5+x)=3$ $\log_2(1-x)(5+x)=3$ $2^3=(1-x)(5+x)$ $x^2+4x+3=0$ $(x+1)(x+3)=0$ **OR/*OF*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-(4)\pm\sqrt{(4)^2-4(1)(3)}}{2(1)}$ $x=\frac{-4\pm\sqrt{4}}{2}$ $x=-1 \quad or/of \quad x=-3$	✓ Log property/*Log wet* $\log_2(1-x)(5+x)=3$ A ✓ Log property/*Log wet* $2^3=(1-x)(5+x)$ CA ✓ Standard form/*Standaard-vorm* $x^2+4x+3=0$ CA ✓ Factors/*Faktore* **OR/*OF*** SF A ✓ $x=-1 \quad or/of \quad x=-3$ CA		(5)	3D
3.3	3.3.1	$z_2=3-2i$	✓ Answer/*Antwoord*	A	(1)	1E
	3.3.2		✓ Diagram	CA	(1)	1D

	3.3.3	$z_1 = 3 + 2i$ $r(mod) = \sqrt{a^2 + b^2}$ $r(mod) = \sqrt{(3)^2 + (2)^2}$ $r(mod) = \sqrt{13}/3{,}61$ $\theta = \tan^{-1}\frac{b}{a}$ $\theta = \tan^{-1}\frac{2}{3}$ $\theta = 33{,}69°$ $z = \sqrt{13}cis33{,}69°$ $z = 3{,}61cis33{,}69°$ OR/*OF* $z = \sqrt{13}(\cos 33{,}69° + i \sin 33{,}69°)$ OR/*OF* $z = \sqrt{13}[33{,}69°$	✓ Modulus ✓ Argument A ✓ z_1 in polar form/*polêre vorm* CA	(3)	3E
3.4	$a(2 - 3ai) - 5 = b(25i - 1) - 2i$ $2a - 3a^2i - 5 = 25bi - b - 2i$ $2a - 5 - 3a^2i = -b + (25b - 2)i$ $2a - 5 = -b$ $b = 5 - 2a$ $-3a^2 = 25b - 2$ $-3a^2 = 25(5 - 2a) - 2$ $3a^2 - 50a + 123 = 0$ $(3a - 41)(a - 3) = 0$ **OR/*OF*** $a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(3)(123)}}{2(3)}$ $a = \frac{50 \pm \sqrt{1024}}{6}$ $a = \frac{41}{3}$ or/*of* $a = 3$ $b = 5 - 2\left(\frac{41}{3}\right)$ or/*of* $b = 5 - 2(3) = -\frac{67}{3} = -1$	✓ M A ✓ Subject/*Onderwerp* CA ✓ Subst/*Vervang* CA ✓ both values of/*beide waardes van a* ✓ both values of/*beide waardes van b* CA	(5)	4M
			[23]	

QUESTION/*VRAAG* 4					
4.1	4.1.1	$y = 3$ $x = 1$	✓✓ Answer/*Antwoord* A (Only mark y = 3)	(2)	1E
	4.1.2	$f: y = a(x-1)^2 + 2$ through/*deur* $A(0;0)$ / $C(2;0)$ $0 = a(0-1)^2 + 2$ $a = -2$ $f(x) = -2(x-1)^2 + 2$	✓ Substitution/*Substitusie* A ✓ $a = -2$ …… CA ✓ Answer/*Antwoord* CA	(3)	2D
	4.1.3	$g: y = \frac{a}{x-1} + 3$ through/*deur* $A(0;0)$ $0 = \frac{a}{0-1} + 3$ $a = 3$ $g(x) = \frac{3}{x-1} + 3$	Don't mark	(0)	
4.2	4.2.1	radius = $5\ m$	✓ $5\ m$ A	(1)	1E
	4.2.2	$x^2 + y^2 = 25$ $(-3)^2 + y^2 = 25$ $y^2 = 16$ $y = \pm 4$ $A(-3;\ -4)\ \ and/en\ \ B(-3;\ 4)$	✓ $x^2 + y^2 = 25$ A ✓SF A ✓ $y = \pm 4$ CA ✓ $A(-3;\ -4)$ CA $B(-3;\ 4)$ **AO $\frac{4}{4}$**	(4)	2D
	4.2.3	$y = \sqrt{25 - x^2}$	✓✓ $y = \sqrt{25 - x^2}$ A	(2)	2D
	4.2.4	$y = -5$	✓ $y = -5$ A	(1)	1E

4.3	4.3.1		For/*Vir* $h(x)$ ✓ Shape/*Vorm* A ✓ y-intercept/*y-afsnit* A ✓ turning point/draaipunt A For/*Vir* $k(x)$ ✓ Shape/*Vorm* A ✓ y-intercept/*y-afsnit* A ✓ x-intercept/*x-afsnit* A	(6)	1D

	4.3.2	$h(x) = (x-3)^2$ $k(x) = -x + 9$ $(x-3)^2 = -x + 9$ $x^2 - 6x + 9 = -x + 9$ $x^2 - 6x + 9 + x - 9 = 0$ $x^2 - 5x = 0$ $x(x-5) = 0$ $x = 0$ or/*of* $x = 5$ CA $y = -(0) + 9$ $y = 9$ $A(0,9)$ $y = -(5) + 9$ $y = 4$ $B(5;4)$	✓$h(x) = k(x)$ A ✓M A If quadratic changed to linear = breakdown CA ✓ $x^2 - 5x = 0$ CA ✓ Factors/Formula/ *Faktore/Formule* CA ✓both x−values/*beide x-waardes* CA ✓ both y-values/*beide y-waardes* **Only coordinates/*Slegs koördinate* A(0 ; 9) B(5 ; 4)** $\frac{6}{6}$.	(6)	3E
	4.3.3	 ✓ Answer/*Antwoord* (A) ✓ Answer/*Antwoord* (B)	CA CA	(2)	1D
	4.3.4	 ✓ Answer/*Antwoord*	CA	(1)	1D
				[28]	

QUESTION/*VRAAG* 5				
5.1	$A = P(1-i)^n$ $A = 390\,099(1-0{,}15)^6$ $A = \text{R } 147\,125{,}65$	✓ Formula/*Formule* A ✓ SF A ✓ Answer/*Antwoord* CA **NPR**	(3)	1D
5.2	$A = P(1+i)^n$ $68000 = P(1+0{,}0892)^{10}$ $P = \frac{68000}{(1+0{,}0892)^{10}}$ $P = \text{R } 28\,935{,}61$	✓ Formula/*Formule* A ✓ SF A ✓ Answer/*Antwoord* CA **NPR**	(3)	2D
5.3	$i_{eff} = \left(1+\frac{i}{m}\right)^m - 1$ $i_{eff} = \left(1+\frac{0{,}085}{12}\right)^{12} - 1$ $i_{eff} = 1{,}08839 - 1$ $i_{eff} = 0{,}08839$ $= 8{,}84\%$	✓ F A ✓ SF A ✓ $i_{eff} = 0{,}08839$ CA ✓ 8,84% CA **NPR**	(4)	2D
5.4	$A = P(1+i)^n$ $A = 80000\left(1+\frac{0{,}075}{4}\right)^{4\times4} \times \left(1+\frac{0{,}092}{12}\right)^{12\times3}$ $A = R141\,768{,}60$ **OR/*OF*** $A = P(1+i)^n$ $A = 80000\left(1+\frac{0{,}075}{4}\right)^{4\times4}$ $A = 107689{,}1465$ $A = 107689{,}1465\left(1+\frac{0{,}092}{12}\right)^{12\times3}$ $A = \text{R141 768,60}$	✓ F A ✓ $\frac{0{,}075}{4}$ A ✓ n= 4 × 4 or/of 16 A ✓ $\frac{0{,}092}{12}$ A ✓ n= 12 × 3 or/of 36 A ✓ R141 768,60 CA A ✓ F ✓ $\frac{0{,}075}{4}$ A ✓ n= 4 × 4 or/of 16 A ✓ $\frac{0{,}092}{12}$ A ✓ n= 12 × 3 or/of 36 A ✓ R141 768,60 CA **NPR**	(6)	3E
			[16]	

QUESTION/*VRAAG* 6

6.1	$f(x) = -2x + 3$ $f(x+h) = -2(x+h) + 3$ $f(x+h) = -2x - 2h + 3$ $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h\to 0} \frac{-2x - 2h + 3 - (-2x + 3)}{h}$ $f'(x) = \lim_{h\to 0} \frac{-2x - 2h + 3 + 2x - 3}{h}$ $f'(x) = \lim_{h\to 0} \frac{-2h}{h}$ $f'(x) = -2$		✓ Definition/*Definisie* A ✓ SF A ✓ S CA ✓ S CA ✓ -2 CA Answer only -1 mark **Penalty: 1 mark for incorrect notation** ***Penaliseer: 1 punt vir verkeerde notasie***	(5)	3D
6.2	6.2.1	$y = 2x^2 - 4x + 6$ $y' = 4x - 4$	✓ $4x$ A ✓ -4 A	(2)	2M
	6.2.2	$y = \sqrt{4x^3} + \frac{1}{x^5} - x$ $y = 2x^{\frac{3}{2}} + x^{-5} - x$ $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 5x^{-6} - 1$ **OR/*OF*** $\frac{dy}{dx} = 3\sqrt{x} - \frac{5}{x^6} - 1$	✓ $2x^{\frac{3}{2}}$ A ✓ x^{-5} A ✓ $3x^{\frac{1}{2}}$ / $3\sqrt{x}$ CA ✓ $-5x^{-6}$ / $-\frac{5}{x^6}$ CA ✓ -1 CA	(5)	2D

6.3	6.3.1	$h(0) = -5(0)^2 + 15(0) + 1 = 1$ $h(2) = -5(2)^2 + 15(2) + 1 = 11$ ave/gem tempo $= \frac{f(b) - f(a)}{b - a}$ $= \frac{11 - 1}{2 - 0}$ $= 5\ m/s$	✓ $h(0) = 1$ & $h(2) = 11$ A ✓ SF CA ✓ Answer/*Antwoord* CA	(3)	2M
				[15]	

QUESTION/*VRAAG* 7				
7.1	12 units/eenhede	Don't mark	(0)	
7.2	$f(1) = (1)^3 - 5(1)^2 - 8(1) + 12 = 0$ $f(1) = 0$ $\therefore (x - 1)$ is a factor/is 'n faktor	✓ SF A ✓ $f(1) = 0$ A	(2)	1M
7.3	Synthetic division/Sintetiese deling $x = 1$ $\begin{array}{\|c\|c\|c\|c\|c\|c\|} \hline x = 1 & 1 & -5 & -8 & 12 & \\ \hline & & 1 & -4 & -12 & \\ \hline & 1 & -4 & -12 & 0 & \\ \hline \end{array}$ $\therefore (x - 1)(x^2 - 4x - 12) = 0$ $(x - 1)(x - 6)(x + 2) = 0$ $\therefore x = 1$ or/*of* $x = 6$ or/*of* $x = -2$ $\therefore C\ (-2; 0)$ and/*en* $E\ (6; 0)$	Accept other methods for finding the roots such as long division./*Aanvaar ander metodes om die wortels te vind soos langdeling.* ✓M A ✓ Factors/*Faktore* CA ✓ x – values/*waardes* CA ✓ $C\ (-2; 0)$ CA ✓ $E\ (6; 0)$ CA Answer only, full marks	(5)	2D

7.4	$f(x) = x^3 - 5x^2 - 8x + 12$			2D
	$f'(x) = 3x^2 - 10x - 8$	✓ $f'(x)$ A		
	$3x^2 - 10x - 8 = 0$	✓ $f'(x) = 0$ A		
	$(3x + 2)(x - 4) = 0$	✓ Factors/*Faktore*		
	OR/*OF*	**OR/*OF***		
	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	SF CA		
	$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(-8)}}{2(3)}$			
	$x = \frac{10 \pm \sqrt{196}}{6}$			
	$\therefore x = -\frac{2}{3}$ or/*of* $x = 4$			
	$y = \left(-\frac{2}{3}\right)^3 - 5\left(-\frac{2}{3}\right)^2 - 8\left(-\frac{2}{3}\right) + 12$	✓Both x-values/ *Beide x-waardes* CA		
	$y = \frac{400}{27} = 14{,}8$	✓ Both y-values/ *Beide y-waardes* CA		
	$y = (4)^3 - 5(4)^2 - 8(4) + 12$	CA		
	$y = -36$	CA		
	Turning points/*Draaipunte*			
	$\therefore A\left(-\frac{2}{3}; 14{,}8\right)$ and/*en* $B\ (4;\ -36)$	✓ $A\left(-\frac{2}{3}; 14{,}8\right)$ ✓ $B\ (4;\ -36)$	(7)	
			[14]	

QUESTION/*VRAAG* 8				
8.1	$h(t) = 16 + 6t - t^2$ $h(0) = 16 + 6(0) - (0)^2$ $= 16\ m$	✓ SF A ✓ $16\ m$ A	(2)	2M
8.2	$h(2) = 16 + 6(2) - (2)^2$ $= 24\ m$	✓ SF A ✓ $24\ m$ A	(2)	1M
8.3	$h'(t) = 6 - 2t$ $6 - 2t = 0$ $2t = 6$ $t = 3$ sec/*sek*	✓ derivative/*afgeleide* A ✓ equating derivative to 0/*stel afgeleide gelyk aan 0* A ✓ 3 sec/*sek* CA	(3)	2D
8.4	$h(3) = 16 + 6(3) - (3)^2$ $= 25\ m$	✓ SF CA ✓ $25\ m$ CA	(2)	1M
			[9]	

QUESTION/*VRAAG* 9					
9.1	9.1.1	$\int (x^2 + y^2)\ dc$ $= x^2c + y^2c + C$	✓ x^2c A ✓ y^2c A ✓ C A	(3)	2E
	9.1.2	$\int \left(2x - \frac{1}{2x} - \sqrt{x} + 4^{3x}\right) dx$ $= \int \left(2x - \frac{1}{2x} - x^{\frac{1}{2}} + 4^{3x}\right) dx$ $= \frac{2x^{1+1}}{1+1} - \frac{1}{2}\ln x - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{4^{3x}}{3\ln 4} + C$ $= \frac{2x^2}{2} - \frac{1}{2}\ln x - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{4^{3x}}{3\ln 4} + +C$ $= x^2 - \frac{1}{2}\ln x - \frac{2}{3}x^{\frac{2}{3}} + \frac{4^{3x}}{3\ln 4} + C$ $= x^2 - \frac{1}{2}\ln x - \frac{2}{3}\sqrt{x^3} + \frac{4^{3x}}{3\ln 4} + C$	✓ $-x^{\frac{1}{2}}$ A ✓ x^2 CA ✓ $-\frac{1}{2}\ln x$ CA ✓ $-\frac{2}{3}x^{\frac{2}{3}}$ / $\frac{2}{3}\sqrt{x^3}$ CA ✓ $\frac{4^{3x}}{3\ln 4} + C$ CA No penalty if C is omitted	(5)	2D

9.2	$f(x) = -x^2 - x + 2$ $\int_{-2}^{1}(-x^2 - x + 2)dx$ $= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 2x\right]_{-2}^{1}$ $= \left(-\frac{(1)^3}{3} - \frac{(1)^2}{2} + 2(1)\right) - \left(-\frac{(-2)^3}{3} - \frac{(-2)^2}{2} + 2(-2)\right)$ $= 4{,}5$ units/*eenhede*^{2}	✓ Area notation must be used for integrals/*Area notasie moet gebruik word vir integrale* A ✓ $-\frac{x^3}{3} - \frac{x^2}{2} + 2x$ A ✓ SF (1) CA ✓ SF (-2) CA ✓ 4,5 or/*of* $\frac{9}{2}$ CA	(5)	2D
			[13]	
			TOTAL/*TOTAAL* 146	