education

Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL SENIOR CERTIFICATE: COMMON TEST
JUNE 2018

TO: THE CHIEF INVIGILATOR OF ALL SCHOOLS OFFERING MATHEMATICS P2: GRADE 12

ERRATA

Please take note of the following change:

Page 4	ERROR	CORRECTION
Question 2 (Instruction)	In the diagram below, a circle ... x-axis at A and B, with A point closest to the origin.	In the diagram below, a circle .... x-axis at A and B, with A point closest to the origin. **The y-axis is a tangent to circle M.**

Kindly ensure that candidates are informed of the Errata.

MR C. KHUMALO
ASSISTANT DIRECTOR
PROVINCIAL EXAMINATION

DATE 2018-June-05

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Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

COMMON TEST

JUNE 2018

MARKS: 125

TIME: 2½ hours

This question paper consists of 6 pages and 1 information sheet.

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INSTRUCTIONS AND INFORMATION

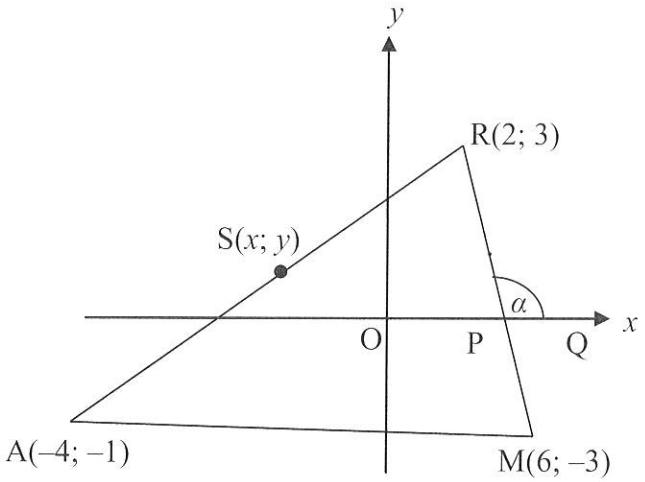
Read the following instructions carefully before answering the questions.

1. This question paper consists of 6 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Write neatly and legibly.

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QUESTION 1

In the diagram alongside, A(–4; –1) R(2; 3) and M (6; –3) are vertices of ΔARM. S(x; y) is the midpoint of RA.
$R\hat{P}Q = \alpha$



1.1 Calculate the co ordinates of S, the midpoint of RA. (2)

1.2 Determine the equation of the straight line RM. (5)

1.3 Calculate the co ordinates of P. (3)

1.4 Calculate the length of RA, rounded off to one decimal digit. (3)

1.5 Show that ΔARM is right-angled. (4)

1.6 Calculate the value of α rounded off to one decimal place. (4)

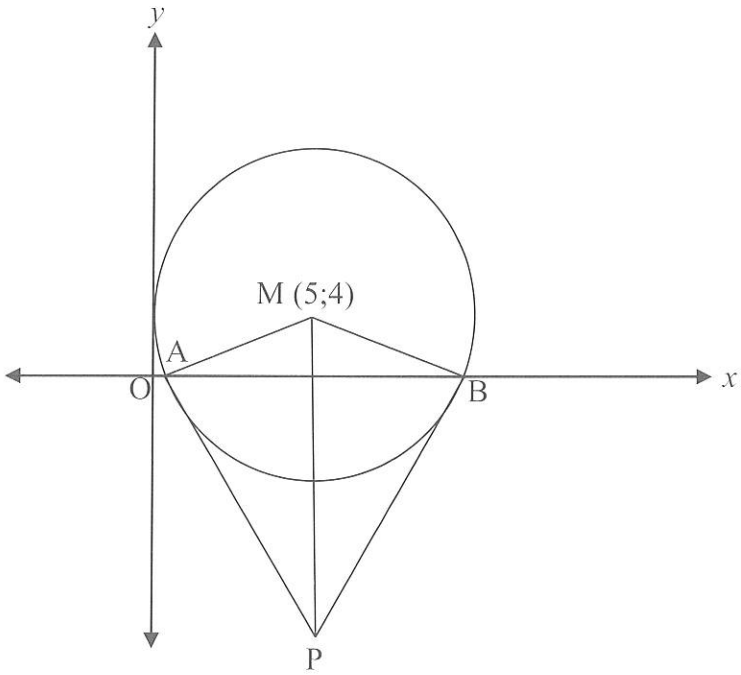
1.7 Calculate the area of ΔARM. (3)

[24]

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QUESTION 2

In the diagram below, a circle with centre M(5;4) and radii AM and MB cut the x-axis at A and B, with A the point closest to the origin.



2.1 Determine the equation of the circle. (3)

2.2 Determine the co-ordinates of A and B. (4)

2.3 Determine the equations of tangents to the circle from a point P with points of contact A and B. (6)

2.4 Calculate the co-ordinates of P. (5)

2.5 Calculate the size of $A\hat{M}B$. (6)
[24]

QUESTION 3

3.1

3.1.1 Show that $\cos(P + Q) + \cos(P - Q) = 2\cos P \cos Q$. (2)

3.1.2 Hence evaluate $\cos 70^\circ + \cos 50^\circ$ without using a calculator if $\cos 10^\circ = 0{,}984$. (7)

3.2 If $\sin 54^\circ = m$, express the following in terms of m, without using a calculator:

3.2.1 $\sin 594^\circ$ (3)

3.2.2 $\cos 18^\circ$ (5)
[17]

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QUESTION 4

4.1 If $\sin 2\theta = \frac{2\sqrt{6}}{5}$ where $\theta \in [45^\circ ; 360^\circ]$ determine, with the aid of a diagram and without using a calculator the value of $\sin \theta$. (6)

4.2 Simplify without using a calculator:

$$\frac{\sin 104^\circ (2\cos^2 15^\circ - 1)}{\tan 38^\circ . \sin 412^\circ}$$ (6)

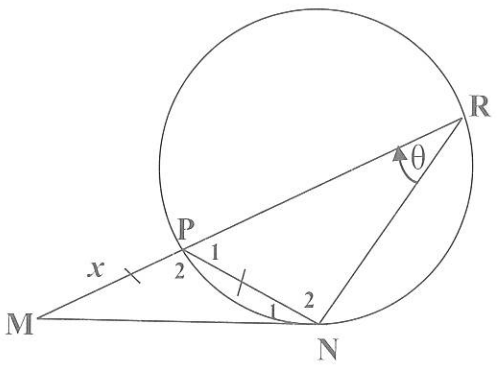
4.3 Prove the identity:

$$\frac{\cos 2x + 1}{\cos 2x \, . \cos^2 x} = \frac{1}{\left(\cos^2 x - \frac{1}{2}\right)}$$ (5)

4.4 Determine the solution of:

$\sin x + \cos x = 1$, *for* $0^\circ \leq x \leq 360^\circ$ (6)

4.5 In the figure alongside, RP is a diameter of the circle. RPM is a straight line and $PM = PN = x$, $P\hat{R}N = \theta$.



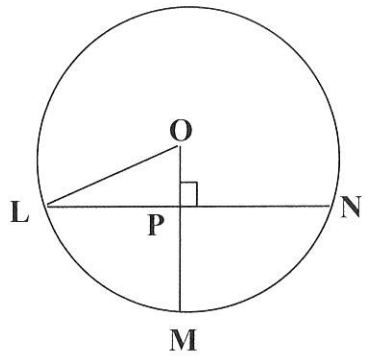
4.5.1 Show that:
$MN^2 = 2x^2(1 + \sin\theta)$ (5)

4.5.2 If $MN = \sqrt{12}$ units and $x = 2$ units, Show, without using a calculator, that $\theta = 30^\circ$ and RP = 4 units. (4)

[32]

QUESTION 5

In the accompanying diagram alongside, LN is a chord of the circle with centre O. OP is drawn perpendicularly onto LN and meets the circle in M.

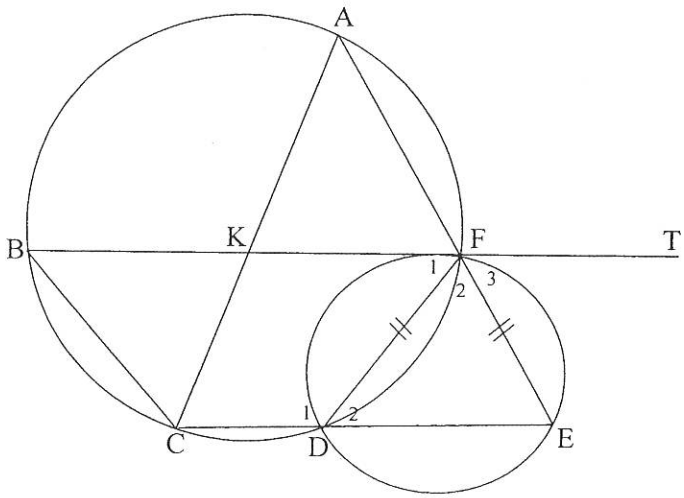


5.1 Prove that $MN^2 = 2\ OM.PM$. (4)

5.2 If OL = 9 units and OP = 1 unit, calculate the length of MN. (2)
[6]

QUESTION 6

In the figure below, two circles cut in points F and D. BFT is a tangent to the smaller circle at F. Straight line AFE is drawn so that FE = FD. CDE is a straight line and chords AC and BF cut at K.



Prove that:

6.1 BT || CE. (4)

6.2 BCEF is a parallelogram. (5)

6.3 AC = BF. (6)

6.4 BF is a diameter if it is given that AF = FE. (7)
[22]

TOTAL MARKS: 125

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INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\,\Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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GRADE 12

MATHEMATICS P2

COMMON TEST

JUNE 2018

ANSWER BOOK

NAME: _____

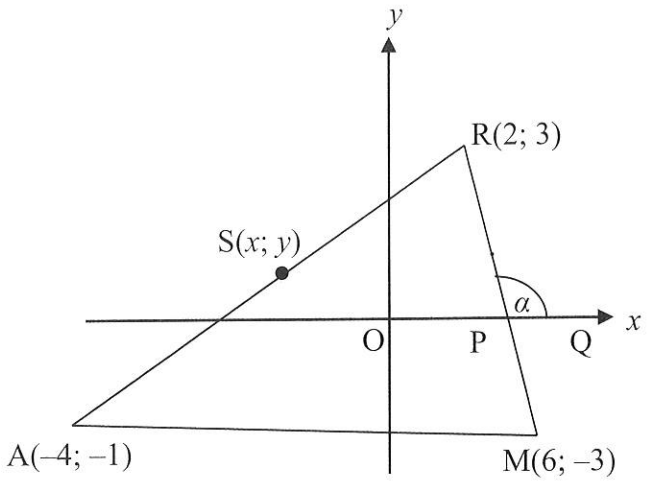
MARKS: 125

TIME: 2½ hours

This answer book consists of 14 pages.

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QUESTION 1



	Solution/Oplossing	Marks / Punte
1.1		(2)

	Solution/Oplossing	Marks / Punte
1.2		(5)

	Solution/Oplossing	Marks / Punte
1.3		(3)

	Solution/Oplossing	Marks / Punte
1.4		(3)

	Solution/Oplossing	Marks / Punte
1.5		(4)

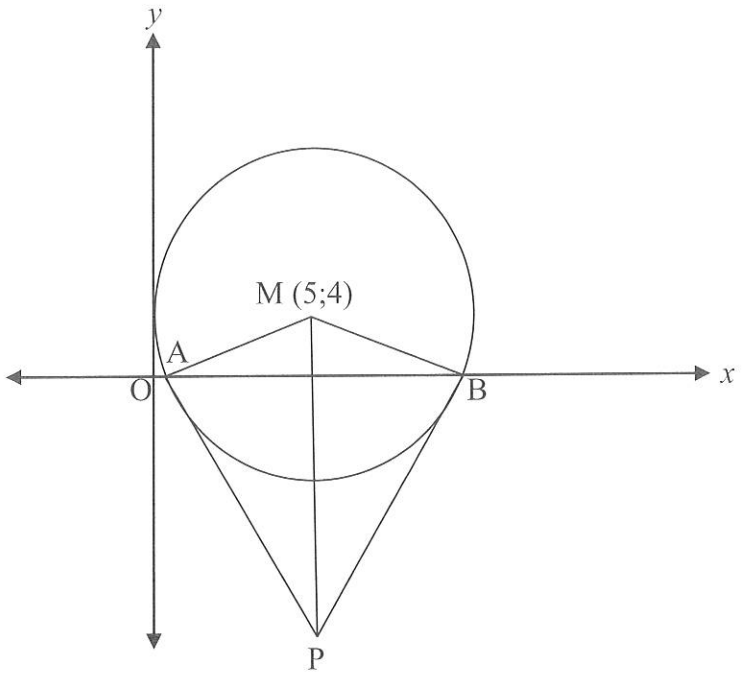
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	Solution/Oplossing	Marks / Punte
1.6		(4)

	Solution/Oplossing	Marks / Punte
1.7		(3)
		[24]

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QUESTION 2



	Solution/Oplossing	Marks / Punte
2.1		(3)

	Solution/Oplossing	Marks / Punte
2.2		(4)

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	Solution/Oplossing	Marks / Punte
2.3		(6)

	Solution/Oplossing	Marks / Punte
2.4		(5)

	Solution/Oplossing	Marks / Punte
2.5		(6)
		[24]

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QUESTION 3

	Solution/Oplossing	Marks / Punte
3.1 3.1.1		(2)

	Solution/Oplossing	Marks / Punte
3.1.2		(7)

	Solution/Oplossing	Marks / Punte
3.2 3.2.1		(3)

	Solution/Oplossing	Marks / Punte
3.2.2		(5)
		[17]

QUESTION 4

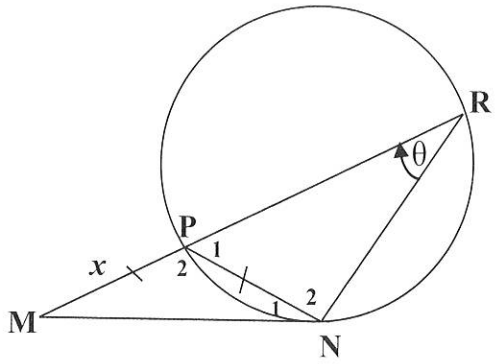
	Solution/Oplossing	Marks / Punte
4.1		(6)

	Solution/Oplossing	Marks / Punte
4.2		(6)

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	Solution/Oplossing	Marks / Punte
4.3		(5)

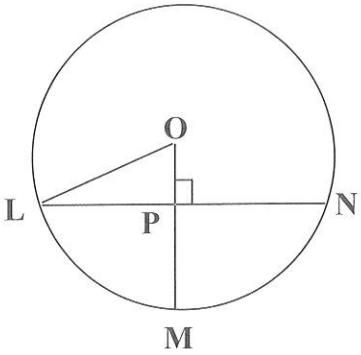
	Solution/Oplossing	Marks / Punte
4.4		(6)



	Solution/Oplossing	Marks / Punte
4.5 4.5.1		(5)

	Solution/Oplossing	Marks / Punte
4.5.2		(4)
		[32]

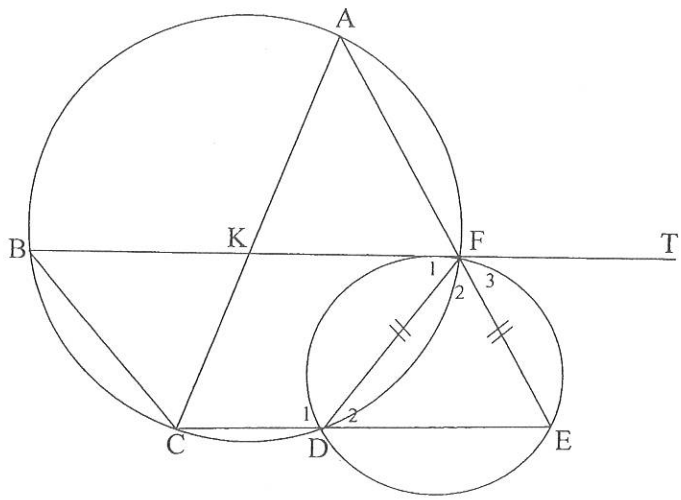
QUESTION 5



	Solution/Oplossing	Marks / Punte
5.1		(4)

	Solution/Oplossing	Marks / Punte
5.2		(2)
		[6]

QUESTION 6



	Solution/Oplossing	Marks / Punte
6.1		(4)

	Solution/Oplossing	Marks / Punte
6.2		(5)

	Solution/Oplossing	Marks / Punte
6.3		(6)

	Solution/Oplossing	Marks / Punte
6.4		(7)
		[22]

TOTAL MARKS: 125

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GRADE 12

MATHEMATICS P2

MEMORANDUM

COMMON TEST

JUNE 2018

MARKS: 125

TIME: 2½ hours

This memorandum consists 9 of pages.

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QUESTION 1

1.1	$S\left(\frac{2+(-4)}{2};\frac{3+(-1)}{2}\right)$ $S(-1;1)$	✓ substitution ✓ answer	(2)
1.2	$m_{RM} = \frac{-3-3}{6-2} = \frac{-6}{4}$ $= \frac{-3}{2}$ $y = \frac{-3}{2}x + c$ $3 = \frac{-3}{2}(2) + c$ $\therefore c = +6$ $\therefore y = \frac{-3}{2}x + 6$	✓ substitution ✓ $M_{RM} = \frac{-3}{2}$ ✓ substituting (2; 3) ✓ C = 6 ✓ answer	(5)
1.3	$\therefore 0 = \frac{-3}{2}x + 6$ $-6 = \frac{-3}{2}x$ $-12 - 3x$ $\therefore x = 4$ P (4; 0)	✓ $y = 0$ ✓ $x = 4$ ✓ answer	(3)
1.4	$RA^2 = (3+1)^2 + (2+4)^2$ $= 4^2 + 6^2$ $= 16 + 36$ $= 52$ $\therefore RA = \sqrt{52}$ $= 7{,}2$	✓ pythagoras ✓ 52 ✓ answer	(3)
1.5	$m_{RA} = \frac{3+1}{2+4}$ $= \frac{4}{6}$ $= \frac{2}{3}$ $M_{RA} \times M_{RM} = \frac{2}{3} \times \frac{-3}{2}$ $= -1$ $\therefore$ Thus RA ⊥ RM $\therefore \Delta$ ARM is right – angled.	✓ substitution ✓ $m_{RA} = \frac{2}{3}$ ✓ product = – 1 ✓ RA ⊥ RM	(4)

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1.6	$\tan\alpha$ = gradient of RM $\tan\alpha = \frac{-3}{2}$ ref angle = 56,3° $\therefore \alpha = 180° - 56,3°$ $= 123,7°$	✓ $\tan\alpha = \frac{-3}{2}$ ✓ ref angle = 56,3° ✓ correct quadrant ✓ answer	(4)
1.7	$RM^2 = (3+3)^2 + (2-6)^2$ $= 36 + 16$ $= 52$ $RM = \sqrt{52}$ $= 7,2$ $RA = 7,2$ Area of Δ ARM $= \frac{1}{2}$ base x height $= \frac{1}{2}$ x 7,2 x 7,2 $= 25,9$ units^2	✓ RM = 7,2 ✓ substitution ✓ answer	(3) **[24]**

QUESTION 2

2.1	$(x-5)^2 + (y-4)^2 = 25$	✓✓ correct substitution ✓ answer	(3)
2.2	$(x-5)^2 + (0+4)^2 = 25$ $(x-5)^2 + 16 = 25$ $(x-5)^2 = 9$ $x - 5 = \pm 3$ $x = 8$ or $x = 2$ A (2; 0) and B (8; 0)	✓ Correct substitution ✓ $x - 5 = \pm 3$ ✓ A (2; 0) ✓ B (8; 0)	(4)

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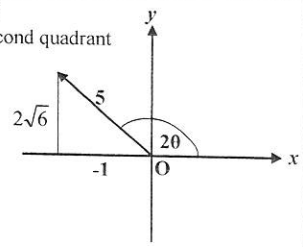
2.3	$m_{MA} = \frac{4}{3}$ $\therefore m_{tangent}$ PA $= \frac{-3}{4}$ through A (2; 0) $\therefore$ Equation of AP $y = \frac{-3}{4}x + \frac{3}{2}$ $m_{MB} = \frac{4}{-3}$ $\therefore y = mx + c$ $y = \frac{3}{4}(8) + c$ $\therefore C = -6$ $\therefore$ equation of PB: $y = \frac{3}{4}x - 6$	✓ $m_{MA} = \frac{4}{3}$ ✓ $\frac{-3}{4}$ ✓ answer ✓ $-\frac{4}{3}$ ✓ C = –6 ✓ answer	(6)
2.4	P : $\frac{3}{4}x - 6 = \frac{-3}{4}x + \frac{3}{2}$ $3x - 24 = -3x + 6$ $6x = 30$ $\therefore x = 5$ $\therefore y = \frac{-9}{4}$ $\therefore P\left(5; \frac{-9}{4}\right)$	✓ equating 2 equations ✓ simplification ✓ $x = 5$ ✓ $y = \frac{-9}{4}$ ✓ answer	(5)
2.5	$\tan\theta = m_{MA} = \frac{4}{3}$ $\therefore \theta = 53,1°$ $\tan\alpha = m_{MB} = \frac{-4}{3}$ $\therefore \alpha = 180° - 53,1°$ $= 126,9°$ $\therefore A\hat{M}B = \alpha - \theta$ … ext $\angle$ of Δ MAB $= 73,8°$	✓ $\tan\theta = \frac{4}{3}$ ✓ $\theta = 53,1°$ ✓ $\tan\alpha = \frac{-4}{3}$ ✓ $\alpha = 126,9°$ ✓ S/R ✓ answer	(6)

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QUESTION 3

3.1	3.1.1	$\text{Cos}(P+Q) = \cos P \cos Q - \sin P \sin Q$ $\text{Cos}(P-Q) = \cos P \cos Q + \sin P \sin Q$ (1) + (2): $\cos P + \cos Q - \sin P \sin Q + \cos P \cos Q + \sin P \sin Q$ $= 2 \cos P \cos Q$	✓ expanding ✓ adding	(2)
	3.1.2	$P + Q = 70°$ $P-Q = 50°$ Adding: $2P = 120°$ $\therefore P = 60°$ Subtracting: $2Q = 20°$ $\therefore \cos 70° + \cos 50° = 2 \cos 60° . \text{Cos } 10°$ $= 2\left(\frac{1}{2}\right)(\cos 10°)$ $= \text{Cos } 10°$ $= 0{,}984$	✓ $P + Q = 70°$ ✓$P-Q = 50°$ ✓$P = 60°$ ✓$Q = 10°$ ✓$2 \cos 60° \cos 10°$ ✓ $\frac{1}{2}$ ✓Answer	(7)
3.2	3.2.1	$\text{Sin } 594°$ $= \sin(360° + 234°)$ $= \sin 234°$ $= \sin(180° + 54°)$ $= -\sin 54°$ $= -m$	 ✓$\sin 234°$ ✓$-\sin 54°$ ✓answer	(3)
	3.2.2	$\cos 18°$ $\sin 54° = \sin(90° - 36°)$ $m = \cos 36°$ $m = 2\cos^2 18° - 1$ $m + 1 = \cos^2 18°$ $\therefore \cos^2 18° = \frac{m+1}{2}$ $\therefore \cos 18° = \sqrt{\frac{m+1}{2}}$	✓$\sin 54° = \cos 36°$ ✓$\cos 2(18°)$ ✓$2\cos^2 18° - 1$ ✓simplifying ✓answer	(5)

QUESTION 4

	4.1	$\theta > 45°$, $2\theta > 90$ $\therefore 2\theta$ is in the second quadrant $r^2 = x^2 + y^2$ $\therefore x^2 = r^2 - y^2$ $= 5^2 - \left(2\sqrt{6}\right)^2$ $= 25 - 24$ $x^2 = 1$ $\therefore x = -1$ $\cos 2\theta = -\frac{1}{5} = 1 - 2\sin^2\theta$ $\sin^2\theta = \frac{3}{5}$ $\therefore \sin\theta = \sqrt{\frac{3}{5}}$	 ✓correct diagram ✓applying Pythagoras ✓$x = -1$ ✓$1-2\sin^2\theta$ ✓simplifying ✓ answer	(6)
	4.2	$\frac{\sin 104°(2\cos^2 15° - 1)}{\tan 38° . \sin^2 412°}$ $= \frac{\sin(180° - 76°)\cos 30°}{\tan 38° . \sin(360° + 52°)}$ $= \frac{\sin 76° . \cos 30°}{\tan 38° . \sin 52°}$ $= \frac{2\sin 38° . \cos 38°\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sin 38°}{\cos 38°}\right)(\cos . 38°)}$ $= \frac{\sqrt{3}\sin 38° \cos 38°}{\sin 38° \cos 38°} = \sqrt{3}$	 ✓$\cos 30°$ ✓$\sin 76$ ✓$\sin 52°$ ✓$\left(\frac{\sqrt{3}}{2}\right)$ ✓$2\sin 38° \cos 38°$ ✓answer	(6)

	4.3	$\frac{\cos 2x+1}{\cos 2x.\cos^2 x}=\frac{1}{\left(\cos^2 x-\frac{1}{2}\right)}$ *LHS* $=\frac{\cos 2x+1}{\cos 2x.\cos^2 x}$ $=\frac{2\cos^2 x-1+1}{\cos 2x\ .\ \cos^2 x}$ $=\frac{2}{\cos 2x}$ $=\frac{2}{2\cos^2 x-1}$ $=\frac{2}{2\left(\cos^2 x-\frac{1}{2}\right)}$ $=\frac{1}{\cos^2 x-\frac{1}{2}}$ = RHS	✓ $2\cos^2 x$ - 1 ✓simplification ✓ $\frac{2}{\cos 2x}$ ✓$2\cos^2 x - 1$ ✓factorising denominator	(5)
	4.4	$\sin x+\cos x=1$ $\sin x=1-\cos x$ $(\sin x)^2=(1-\cos x)^2$ $\sin^2 x=1-2\cos x+\cos^2 x$ $1-\cos^2 x=1-2\cos x+\cos^2 x$ $-2\cos^2 x+2\cos x=0$ $-2\cos x(\cos-1)=0$ $\therefore \cos x=0$ $\therefore x=0^o$ *OR* 270^o $\therefore x\in[0^o;90^o]$	✓squaring both sides ✓simplifying ✓factorising ✓0^o ✓Rejecting 270^o ✓90^o	(6)
	4.5.1	$P\hat{N}R=90^o$ ($\angle$ *in semicircle*) $N\hat{P}R = 90^o-\theta$ $M\hat{P}N = 90^o+\theta$.... *straight line* $R\hat{P}M$ $MN^2 = MP^2 + PN^2 - 2MP\ .\ PN\ .\ \cos M\hat{P}N$ $= x^2 + x^2 - 2x^2\cos(90^o+\theta)$ $=2x^2 - 2x^2(-\sin\theta)$ $= 2x^2(1+\sin\theta)$	✓ S/R ✓ S ✓ S/R ✓ cos rule ✓ simplifying	(5)

	4.5.2	$\left(\sqrt{12}\right)^2 = 2(2)^2(1+\sin\theta)$ $12 = 8(1+\sin\theta)$ $\frac{3}{2}=1+\sin\theta$ $\therefore \sin\theta=\frac{1}{2}$ $\therefore \theta = 30^o$ $\therefore \sin\theta=\frac{PN}{RP}=\frac{2}{RP}=\frac{1}{2}$ $\therefore RP = 4$ *units*	✓ subst in 4.5.1 ✓ simplifying ✓ $\sin\theta=\frac{1}{2}$ ✓ $\sin\theta=\frac{PN}{RP}=\frac{1}{2}$	(4)

QUESTION 5

5.1	$MN^2 = PM^2 + PN^2$ ... *pythagoras*, Δ *PMN* $= PM^2 + (ON^2 - OP^2)$ ... *pythagoras*, Δ *PON* $= PM^2 + OM^2 - (OM - PM)^2$ ... $ON = OM$ $= PM^2 + OM^2 - OM^2 + 2.OM.PM - PM^2$ $= 2\ OM\ .\ PM$	✓ S/R ✓ S/R ✓ S ✓ S	(4)
5.2	$MN^2 = 2.(9)(8)$ ... OL = OM = 144 $\therefore$ MN = 12 units	✓ S/R ✓ answer	(2)
			[6]

QUESTION 6

6.1	$\hat{F} = \hat{E}$ .... tan BF, chord DF $= \hat{D}_2$ ... FD = FE $\therefore$ BFT // CE ... alt $\angle$'*s* proved equal	✓ S ✓R ✓ S ✓ R	(4)
6.2	$\hat{F}_3 = \hat{D}_2$ ... tan BT, chord EF $= \hat{B}$ ... ext $\angle$ cyclic quad BCDF $\therefore$ BC // FE .... corres $\angle$'s proved equal $\therefore$ BCEF is a parallelogram	✓ S ✓ R ✓ S ✓ S ✓ R	(5)
6.3	$\hat{E} = \hat{B}$ ... opp $\angle$'*s* of a parm $= \hat{A}$ ... subtends the same arc CF $\therefore$ AC = CE ... $\angle$'*s* opposite to equal sides = BF ... opp. Sides of a parm	✓ S/R ✓ S ✓R ✓ S ✓R ✓ R	(6)

6.4	$KF = \frac{1}{2}CE$ ... AF = FE; KF // CE
	$= \frac{1}{2}BF$ ... from 7.3
	∴ k is the mid point of BF
	AK = KC ... KF = FE, KF // CE
	∴ K is the midpoint of AC
	∴ BK = KF = AK = KC = radius
	∴ BF is the diameter
		✓ S/R
		✓ S
		✓ S
		✓ S/R
		✓ S
		✓ S
			(7)

			[22]

TOTAL MARKS: 125

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