GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PROVINCIAL EXAMINATION

NOVEMBER 2021

GRADE 11

MARKING GUIDELINES

MATHEMATICS (PAPER 2)

17 pages

EXEMPLAR

INSTRUCTIONS AND INFORMATION:

A – Accuracy

C.A. – Continued Accuracy (follow up)

S – Statement

R – Reason

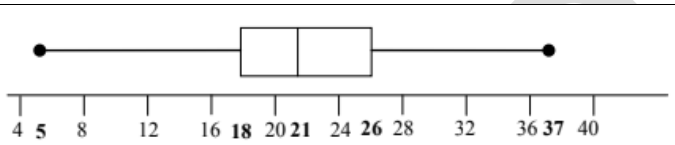
S and R – Statement and Reason

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- It is unacceptable to adopt values/answers in order to solve a problem.

EXEMPLAR

QUESTION 1

1.1	Mean $= \frac{\sum_{i=1}^{n} x_i}{n} = \frac{408}{19} = 21{,}47$	✓ answer	(1)
1.2	Standard deviation = 7,81	✓✓ answer	(2)
1.3	The one standard deviation limits are $(\bar{x} - 1\sigma\ ;\ \bar{x} + 1\sigma)$ = (21,47 – 7,81; 21,47 + 7,81) = (13,66 ; 29,28) ∴ 13 people lie within 1 standard deviation of the mean	✓ interval ✓ 13 people	(2)
1.4	5 12 13 15 18 18 18 19 20 21 21 22 23 23 26 29 33 35 37 IQR = 26 – 18 = 8	✓ $Q_1 = 18$ ✓ $Q_3 = 26$ ✓ IQR = 8	(3)
1.5		✓✓ box ✓ whiskers	(3)
1.6	There is a marked difference between the lowest value (5) and the next lowest value (12) whilst the differences between all other data points are within at most 3 values > ∴ 5 is an outlier	✓ reason ✓ 5 is an outlier	(2)
			[13]

EXEMPLAR

QUESTION 2

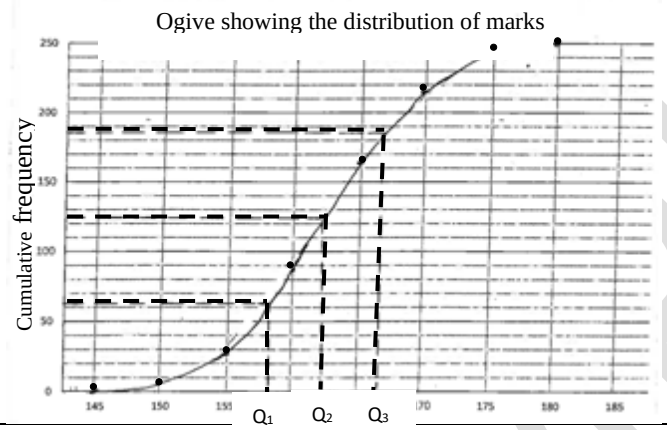
2.1

Height (cm)	Number of pupils (f)	Cumulative frequency
$145 \leq \times < 150$	6	6
$150 \leq \times < 155$	23	29
$155 \leq \times < 160$	60	**89**
$160 \leq \times < 165$	74	163
$165 \leq \times < 170$	52	**215**
$170 \leq \times < 175$	32	**247**
$175 \leq \times < 180$	3	250
Total	**250**	

✓ 89
✓ 215
✓ 247

(3)

2.2



✓ Shape
✓ anchor on *x*- axis
✓ plotting

(3)

2.3

$Q_2 \approx 162$

$Q_1 \approx 158$

$Q_3 \approx 167$

$\therefore$ IQR ≈ 162

✓
✓
✓
✓

(4)

2.4

$165 \leq \times < 170$

✓ answer (1)

2.5

$\frac{35}{250} = 14\%$

✓ answer (1)

2.6

$\sigma = 6{,}43$

✓✓ answer (2)

[14]

EXEMPLAR

QUESTION 3

3.1	E $\left(\frac{3+12}{2}\ ;\ \frac{0+3}{2}\right)$ $=\left(7\frac{1}{2}\ ;\ 1\frac{1}{2}\right)$	✓ substitution into midpoint formula ✓ Answer	(2)
3.2	$M_{BC}=\frac{3-0}{12-3}$ $=\frac{1}{3}$	✓ substitution into gradient formula ✓ Answer	(2)
3.3	$\tan\theta = m_{BC} = \frac{1}{3}$ $\theta\tan^{-1}\left(\frac{1}{3}\right) = 18{,}43^\circ$	✓ $\tan\theta = m_{BC}$ ✓ answer	(2)
3.4	$M_{AD} = M_{BC} = \frac{1}{3}$ AD||BC, equal gradients $M_{AB} = \frac{6-0}{1-3} = -3$ $\therefore M_{AD} \times M_{AB} = \frac{1}{3} \times -3 = -1$ $\therefore$ AD $\perp$ AB	✓ $M_{AD} = \frac{1}{3}$ ✓ $M_{AB} = -3$ ✓ $M_{AD} \times M_{AB} = -1$	(3)
3.5	Inclination of new line = $45^\circ + 18{,}43^\circ = 63{,}43^\circ$ $\therefore \tan 63{,}43^\circ = 2 = m_{line}$ $\therefore y - 6 = 2(x - 1)$ $y = 2x + 4$	✓ $18{,}43^\circ$ ✓ $63{,}43^\circ$ ✓ $m = 2$ ✓ equation	(4)
			[13]

EXEMPLAR

QUESTION 4

4.1	$M_{QP} = M_{OS} = 6$ QP||OS, equal gradients $y - 17 = 6(x + 3)$ $y = 6x + 35$	✓ $M_{QP} = 6$ ✓ equation	(2)
4.2	$6x + 35 = -x$ $7x = -35$ $x = -5$ $y = -(-5) - 5$ $\therefore Q(-5;5)$	✓ setting up equation ✓ $x = -5$ ✓ coordinates of Q	(3)
4.3	$OQ^2 = (-5 - 0)^2 + (5 - 0)^2$ $= 50$ $OQ = \sqrt{50} = 5\sqrt{2}$ units	✓ substitution into distance formula ✓ $5\sqrt{2}$	(2)
4.4	$M_{os} = 6$ $\therefore$ inclination of OS is $-\tan^{-1}(4) = 80{,}54°$ $M_{OQ} = -1$ $\therefore$ inclination of QO is $180° - \tan^{-1}(1) = 135°$ $\alpha = 135° - 80{,}54°$ … $= 54{,}46°$	✓ 80,54° ✓ 135° ✓ 54,46°	(3)
4.5	$QS^2 = OS^2 + OQ^2 - 2OS.OQ.\cos\alpha$ $= 148 + 50 - 2(\sqrt{148})(\sqrt{50}.\cos 54{,}46°)$ QS = 9,90 units	✓ correct use of cosine rule ✓ substitution into formula ✓ 9,90	(3)
			[13]

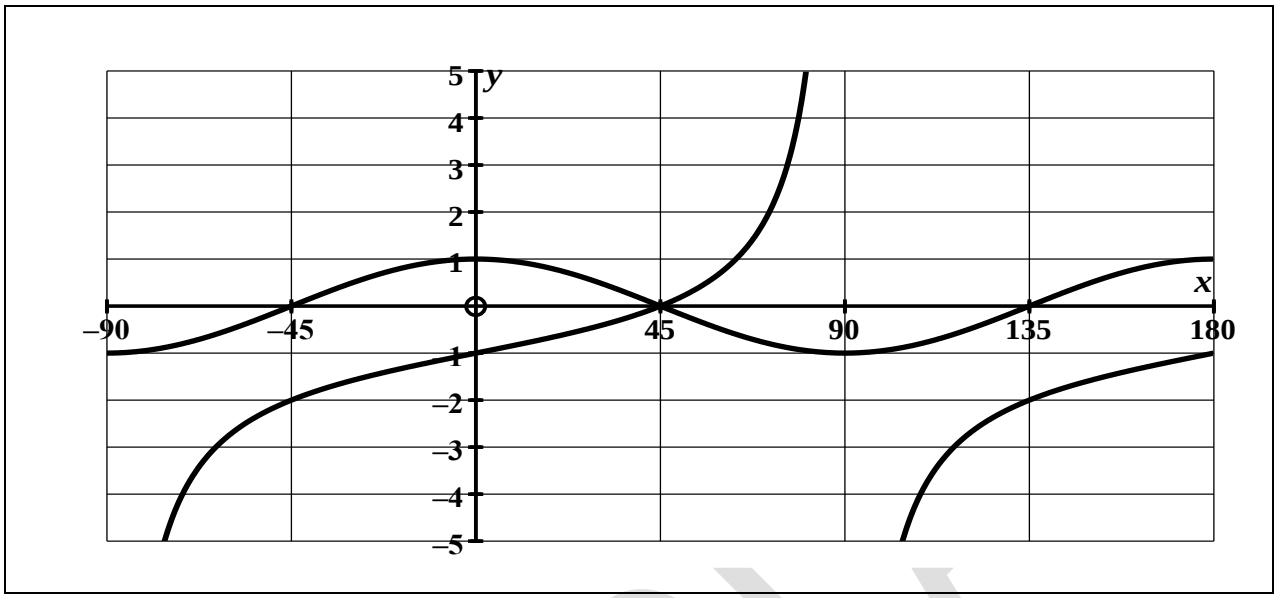
EXEMPLAR

QUESTION 5

5.1	5.1.1	= cos 62° = *k*	✓✓ Answer	(2)
	5.1.2	= –cos 62° = –*k*	✓✓ Answer	(2)
5.2	= -tan *x* . cos *x* = $\frac{\sin x}{\cos x} \times \frac{\cos x}{\sin x}$ –sin *x* = 1		✓ tan *x* . cos *x* ✓ –sin *x* ✓ $\frac{\sin x}{\cos x}$ ✓ 1	(4)
5.3	$\frac{\cos 50° - \cos \theta}{\sin 40 + \cos \theta} = \frac{-(\cos \theta + \cos 50°)}{\cos \theta + \sin 40°}$ cos 50° = sin 40° $\therefore = -1$		✓✓ $\frac{\cos 50° - \cos \theta}{\sin 40 + \cos \theta}$ ✓✓ –(cos θ + cos 50°) ✓ answer	(5)
5.4	$= \sqrt{2^{2\sin 20°}.2^{3.-\tan 45°}}$ $= \sqrt{2^{2.\frac{1}{2}}.2^{-3}}$ $= \sqrt{\frac{1}{4}}$ $= \frac{1}{2}$		✓ $\sqrt{2^{2\sin 20°}.2^{3.-\tan 45°}}$ ✓ $\sqrt{2^{2.\frac{1}{2}}.2^{-3}}$ ✓ $\frac{1}{2}$	(3)
5.5	5.5.1	$\sin^2 x = \frac{3}{4}$ $\sin x = \pm \frac{\sqrt{3}}{2}$	✓ answer	(1)
	5.5.2	Reference angle = 60° 60°; 120°; 240°	✓✓✓ answers	(3)
				[20]

EXEMPLAR

QUESTION 6

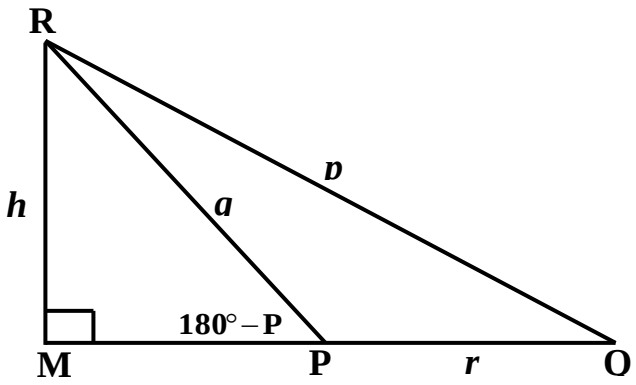


6.1	$f(x)=\tan x-1$ $g(x)=\cos 2x$		✓ asymptotes ✓ intercepts with axes ✓ shape ✓ period ✓ shape with open dot at $x=-90°$	(5)
6.2	6.2.1	$\cos 2x+1=\tan x$ $\therefore \cos 2x=\tan x-1$ $\therefore x=45°$	✓ $x=45°$	(1)
	6.2.2	$2\cos^2 x=1$ $\therefore 2\cos^2 x-1=0$ $\therefore \cos 2x=0$ $\therefore x=\pm 45°$ or $x=135°$	✓ $\cos 2x=0$ ✓ solutions	(2)
	6.2.3	$-90° < x < -45°$ of $90° < x < 135°$	✓$-90° < x < -45°$ ✓ $90° < x < 135°$	(2)
6.3	$h(x)=\cos 2(x-30^\circ)$		✓ $h(x)=\cos 2(x-30^\circ)$	(1)
				[11]

EXEMPLAR

QUESTION 7

7.1



Construction: Draw RM ⊥ PQ or QP produced

$RM = p\sin Q$

$CM = q\sin(180° - P)$

$= q\sin P$

$\therefore \quad p\sin Q = q\sin P$

Dividing by sinP.sinQ gives

$$\frac{p}{\sin P} = \frac{q}{\sin Q}$$

✓ construction

✓ $RM = p\sin Q$

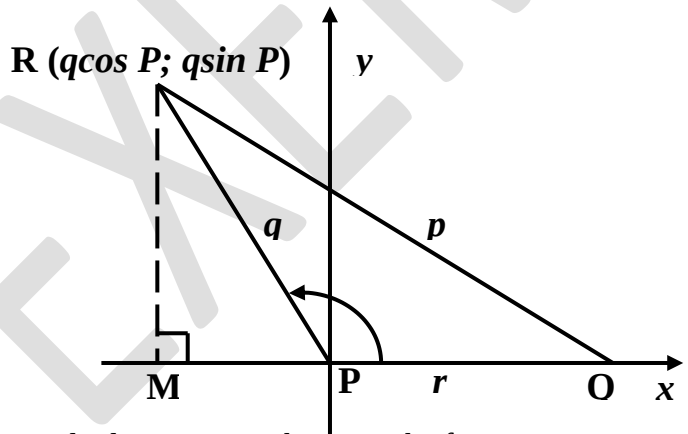
✓ $RM = q\sin(180° - P)$

✓ $\therefore \quad p\sin Q = q\sin P$

✓ Dividing by sinP.sinQ

OR

OR



✓ **R(*q*cosP; *q*sinP)**

P is in standard position as shown in the figure.

Construction: Draw RM ⊥ PQ or QP produced

$RM = q\sin P = p\sin Q$

Dividing by sinP.sinQ we then have

$$\frac{p}{\sin P} = \frac{q}{\sin Q}$$

✓ construction

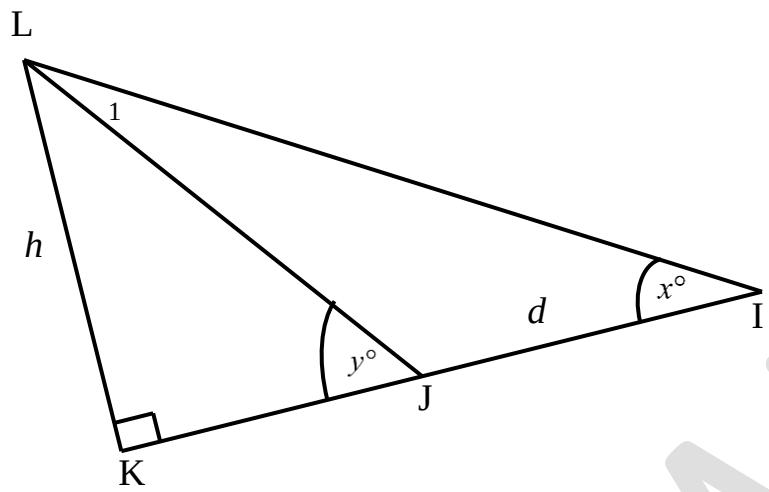
✓ $RM = q\sin P$

✓ $RM = p\sin Q$

✓ Dividing by sinP.sinQ

(5)

EXEMPLAR

7.2				
7.2	7.2.1	$In\Delta\ LKJ$: $\frac{h}{LJ}=\sin y$ $\therefore h=LJ\sin y \quad ---\quad (1)$ $In\Delta\ DAB$: $\frac{LJ}{\sin x}=\frac{d}{\sin L_1}$ $but\ \ \hat{L}_1=y-x$ $\therefore\ LJ=\frac{d\sin x}{\sin(y-x)} \quad ---\quad (2)$ subst.(2) into (1) : $h=\frac{d\sin x\sin y}{\sin(y-x)}$	✓ h subt of formula ✓ sine rule ✓ $\hat{L}_1=y-x$ ✓ LJ ✓ substitution	(5)
	7.2.2	$h=\frac{70.\sin 12°.\sin 30°}{\sin 18°}$ $=23,55\,m$	✓ substitution ✓ answer	(2)
				[12]

EXEMPLAR

QUESTION 8

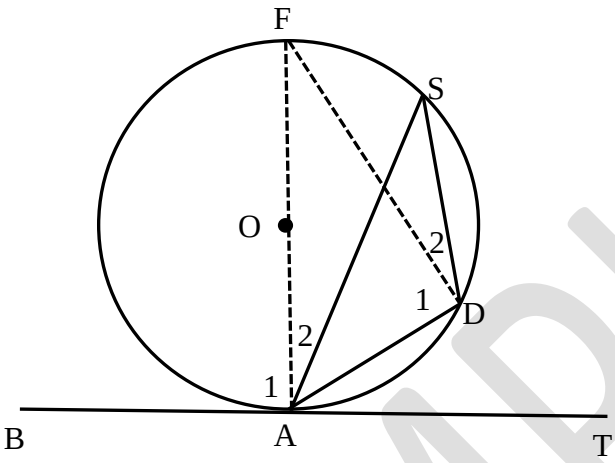
8.1	Volume of cylinder: $V_1 = \pi r^2 h$ $= \pi(12)^2.(19)$ $= 2736\ \pi\ m^3$ Volume of hemisphere (half sphere): $V_2 = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ *Total Volume* $= V_1 + V_2$ $= \frac{1}{2}\left[\left(\frac{4}{3}\pi r^3\right)(12)^3\right]$ $= 1152\ \pi\, m^3$ *Total Volume* $= V_1 + V_2$ $= 2736\pi + 1152\pi$ $= 3888\pi m^3$	✓ Substitution ✓ answer ✓ $\frac{1}{2}$ ✓ Substitution ✓ answer ✓ Total Volume in π (***5 out of 6 if numerical value of π is used***)	(6)

EXEMPLAR

8.2	Surface area of cylinder: $A_1 = 2\pi r^2 + 2\pi rh$ $= 2\pi(12)^2 + 2\pi(12)(19)$ $= 2337{,}34\ m^2$ Surface area of hemisphere (half sphere): $A_2 = \frac{1}{2}(4\pi r^2)$ $= \frac{1}{2}\left[(4\pi r^2)(12)^2\right]$ $= 452.39\ m^2$ *Total Surface Area* $= A_1 + A_2$ $= 2337{,}34 + 452.39$ $= 685.73\ m^2 \approx 686\ m^2$	✓ Substitution ✓ answer ✓ Substitution ✓ answer ✓ Total area ✓ correct rounded off answer	(6)
			[12]

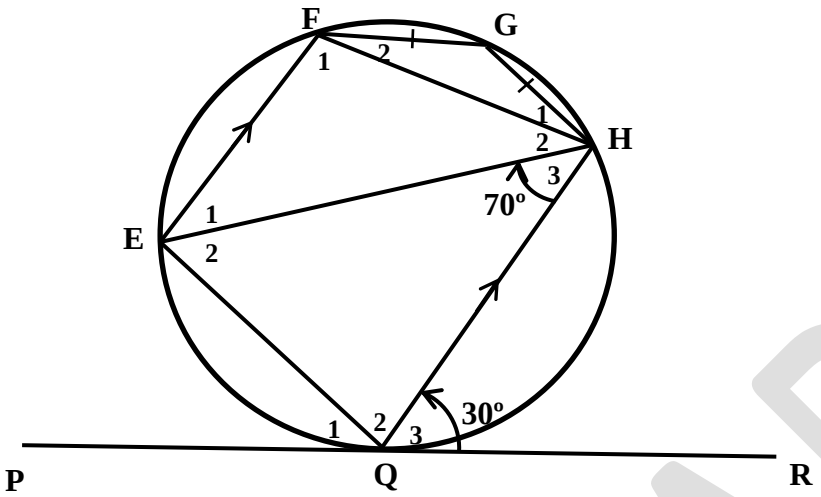
EXEMPLAR

QUESTION 9

9.1	9.1.1	The opposite angles of a cyclic quadrilateral are ***supplementary.***	✓ answer	(1)
	9.1.2	The angle between a tangent and a chord is ***equal to an angle in an alternate segment.***	✓ answer	(1)
9.2	 Construct AF(diameter) and FD (chord) $\hat{D}_1 = 90^\circ$ *($\angle$ in a semi-circle)* $\hat{A}_1 = 90^\circ$ *(rad $\angle$ tan)* $\hat{A}_2 = \hat{D}_2$ *($\angle$s in same seg.)* $\hat{A}_1 + \hat{A}_2 = \hat{D}_1 + \hat{D}_2$ $\therefore B\hat{A}S = \hat{D}$	✓ Construction ✓ Statement ✓ Reason ✓ Statement and Reason ✓ Statement and Reason ✓ Conclusion	(6)

EXEMPLAR

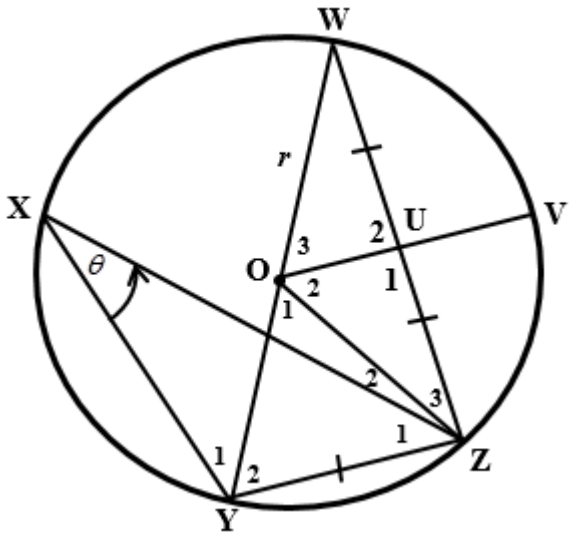
9.3



	9.3.1	$P\hat{Q}E = \hat{H}_3 = 70°$ *(tan – chord theorem)* $\therefore \hat{Q}_1 = 70°$	✓ tan – chord teorem ✓ $\hat{Q}_1 = 70°$	(2)
	9.3.2	$\hat{E}_1 = 70°$ *(alt.∠s, EF // QH)* $\hat{G} = 180° - \hat{E}_1$ *(opp.∠s cyclic quad EFGH)* $= 180° - 70°$ $\hat{G} = 110°$	✓ $\hat{E}_1 = 70°$ ✓ *alts ∠s =* ✓ $\hat{G} = 180° - \hat{E}_1$ ✓ **R**	(4)
	9.3.3	$\hat{Q}_2 = 180° - (\hat{Q}_1 + \hat{Q}_3)$ *(Str.line PQR)* $= 180° - (70° + 30°)$ $= 80°$ $E\hat{F}H = 180° - \hat{Q}_2$ *(opp.∠s cyc.quad EFHQ)* $\hat{F}_1 = 180° - 80°$ $= 100°$	✓ $180° - (Q_1 + \hat{Q}_3)$ ✓ $\hat{Q}_2 = 80°$ ✓ *opp ∠s of cyc quad* ✓ $\hat{F}_1 = 100°$	(4)
				[18]

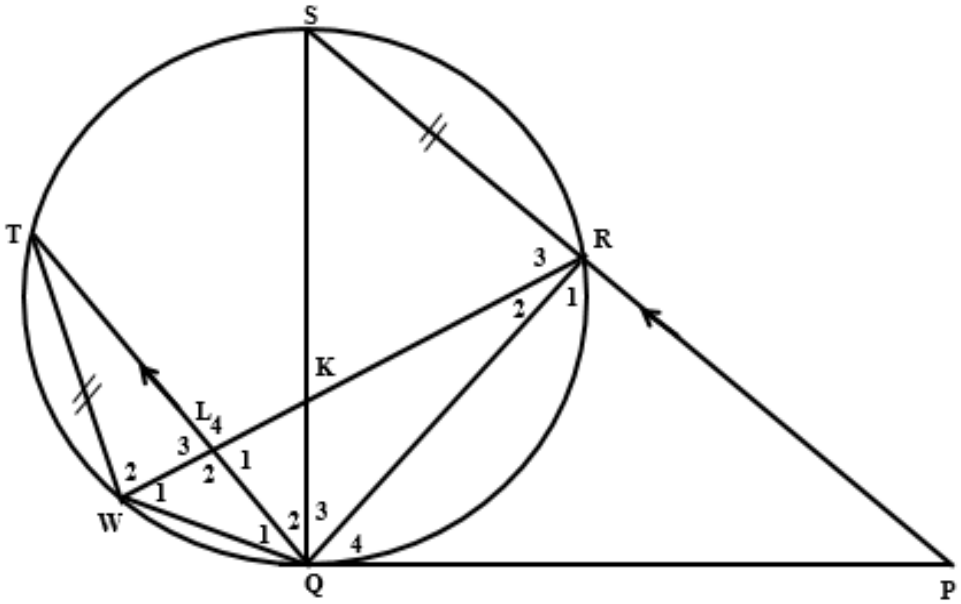
EXEMPLAR

QUESTION 10

10.1				
	10.1.1	$\angle O_1 = 2\theta$ *(∠ at the centre= 2x the ∠ at the circumference)*	✓statement✓reason	(2)
	10.1.2	$\angle W = \theta$ *(∠s in the same seg)*	✓statement✓reason	(2)
	10.1.3	$\angle Z = 90^\circ$ *(∠ in a semi-circle)* $\angle Y_2 = 90 - \theta$ (sum of ∠s in a Δ)	✓statement & reason ✓statement & reason	(2)
10.2	$WY^2 = WZ^2 + YZ^2$ *(Pythagoras theorem)* $YZ = WU = UZ$ *(given)* $\therefore OW = OY = r$ *(given)* $\Rightarrow YZ = \sqrt{\frac{4r^2}{5}} = \frac{4r}{\sqrt{5}}$		✓statement & reason ✓statement & reason ✓statement & reason $YZ = \sqrt{\frac{4r^2}{5}}$ **OR** $YZ = \frac{4r}{\sqrt{5}}$ ✓	(4)
10.3	OW=OV= r =OU+OV *(radii)* $r^2 = 3^2 + 4^2$ *(Pythagoras theorem)* $r = 5$ *cm* $\therefore UV=5-2$ $=3$ *cm*		✓statement✓reason $r^2 = 3^2 + 4^2$ ✓ r =5 cm✓ ✓answer	(2)
				[12]

EXEMPLAR

QUESTION 11



11.1	$\hat{S} = \hat{Q}_2 = x$ *(alt angles TQ||PS)* $\hat{S} = \hat{W}_1 = x$ *(∠s in same segment)* $\hat{S} = \hat{Q}_4 = x$ *(∠ between tangent and chord)*		✓ statement & reason ✓ statement & Reason ✓ statement & Reason	(3)
11.2	11.2.1	$\hat{R}_1 = \hat{S} + \hat{Q}_3$ *(ext. ∠ of Δ* $\hat{L}_3 = \hat{W}_1 + \hat{Q}_1$ *(ext. ∠ of Δ)* Since $\hat{S} = \hat{W}_1 = x$ *(∠s in same segment)* $\hat{Q}_1 = \hat{Q}_3$ *(= chord subt. = ∠)* $\therefore \hat{R}_1 = \hat{L}_3$	✓ statement & reason ✓ statement & reason ✓ statement ✓ conclusion	(4)

EXEMPLAR

	11.2.2	$\hat{W}_1 = \hat{Q}_4$ *(tan-chord theorem)* since $\hat{Q}_1 = \hat{Q}_3$ *(= chord subt. = ∠)* $\Rightarrow \hat{Q}_4 + \hat{Q}_3 = \hat{W}_1 + \hat{Q}_1$ also $\hat{L}_3 = \hat{W}_1 + \hat{Q}_1$ *(ext. ∠ of Δ)* $= \hat{R}_3$ *(corr. ∠s ,QT ||RS)* $\therefore \hat{R}_3 = \hat{Q}_4 + \hat{Q}_3$ PRKQ is a cyclic quad (*ext. ∠ = int. opp. ∠)* ***Alternatively:*** $\hat{Q}_1 + \hat{Q}_3 = \hat{Q}_3 + \hat{S}$ **OR** $\hat{W}_1 = \hat{Q}_4$ *(tan-chord theorem)* $\hat{Q}_1 = \hat{Q}_3$ *(= chord subt. = ∠)* $\therefore \hat{Q}_4 + \hat{Q}_3 = \hat{W}_1 + \hat{Q}_1$	✓ statement ✓ reason ✓ statement & reason ✓ statement & reason ✓ statement & reason	(5)
				[12]
			TOTAL:	**150**

EXEMPLAR