education

Department of Education
FREE STATE PROVINCE

GRADE 12

MATHEMATICS

MARKING GUIDELINE

MOCK EXAM

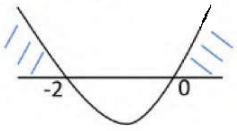
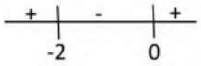
SEPTEMBER EXAM PAPER 1

25 AUGUST 2023

MARKS: 150

This marking guideline consists of 14 pages.

QUESTION 1

1.1.1	$x(x+2)=0$ $x=0$ OR/OF $x=-2$	✓$x=0$ ✓ x $=-2$ (2)
1.1.2	$x^2+2x\geq 0$ $x(x+2)\geq 0$  OR/OF  $\therefore x>0$	✓$x^2+2x\geq 0$ ✓graph/grafiek/ critical pts ✓ $x>0$ If $x\geq 0$ only 1 mark Indien $x\geq 0$ slegs 1 punt (3)
1.2	$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-7)}}{2(2)}$ $=\frac{3\pm\sqrt{65}}{4}$ $x=2{,}77 \quad OR/OF \quad x=-1{,}27$	✓ substitution into correct formula/ *substitusie in korrekte formule* ✓ simplify ✓2,77 ✓ -1,27 If left in surd form only 2 marks. Indien in wortelvorm, slegs 2 punte. (4)
1.3.1	$k^2+5k-14=0$ $(k-2)(k+7)=0$ $k=2$ or $k=-7$	✓ multiplying ✓ factorising ✓ both solutions (3)
1.3.2	$\sqrt{x+5}=2$ or $\sqrt{x+5}=-7$ $x+5=4$ or invalid/ no solution $x=-1$	✓ substitution ✓ identifying invalid ✓ value of x
1.4	$x-2y-3=0$ ……. (1) $4x^2-5xy+y^2=0$ ….. (2) From eq (1): $x=2y+3$ …(3) Subst. (3) into (2): $\therefore 4(2y+3)^2-5y(2y+3)+y^2=0$ $\therefore 4(4y^2+12y+9)-5y(2y+3)+y^2=0$ $\therefore 16y^2+48y+36-10y^2-15y+y^2=0$ $\therefore 7y^2+33y+36=0$ $\therefore (7y+12)(y+3)=0$ $\therefore y=\frac{-12}{7}$ or $y=-3$ $\therefore x=\frac{-3}{7}$ or $x=-3$	✓ equation (3) ✓ substitution ✓ simplification ✓ standard form ✓ factors ✓ both y values ✓ both x-values (7)

1.5	For real roots : $4-20k \geq 0$ $\therefore -20k \geq -4$ $\therefore k \leq \frac{1}{5}$	✓ $4-20k \geq 0$ ✓ Answer (2)
		[24]

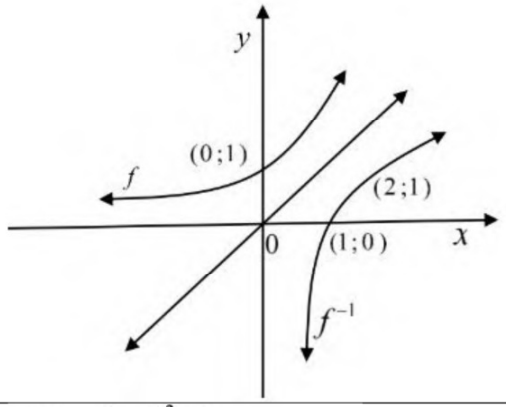
QUESTION 2

2.1.1	1 ; −5 ; −13; −23 −6 −8 −10 −2 −2 $2a=-2$ $3a+b=-6$ $a+b+c=1$ $a=-1$ $3(-1)+b=5$ $-1-3+c=1$ $b=-3$ $c=5$ $T_n=-n^2-3n+5$	✓ Value of a ✓ Value of b ✓ Value of c ✓ General term (3)
2.1.2	$-n^2-3n+5=-643$ $n^2+3n-648=0$ $(n+27)(n-24)=0$ $n=-27$ or $n=24$ $\therefore T_{24}=-643$	✓ equating ✓ factors ✓ answer (3)
2.2.1	$3k-(2k+1)=5k-5-3k$ $k-1=2k-5$ $k=4$	✓ Difference ✓ answer (2)
2.2.2	9,12,15 $T_n=3n+6$ $\sum_{n=1}^{20}(3n+6)$	✓ Sequence ✓ General term ✓ Sigma Notation (3)

2.3.1	$r = \frac{4x^4}{8x^3}$ $r = \frac{x}{2}$ $-1 < r < 1$ $-1 < \frac{x}{2} < 1$ $-2 < x < 2$	✓ value of r ✓ Condition for converging series ✓ answer (3)
2.3.2	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{8x^2}{1-\frac{x}{2}}$ $\frac{8}{3}\left(1-\frac{x}{2}\right) = 8x^2$ $\frac{8}{3} - \frac{8x}{6} = 8x^2$ $16 - 8x = 48x^2$ $48x^2 + 8x - 16 = 0$ $6x^2 + x - 2 = 0$ $(3x+2)(2x-1) = 0$ $x = -\frac{2}{3}$ or $x = \frac{1}{2}$	✓ Substitution into the correct formula ✓ Standard form ✓ Factors ✓ Answers (4)
2.4.1	$a + 2d = ar$ $1 + 2d = 1r$ $d = \frac{r-1}{2}$ ⟶ $a + 12d = ar^2$ $1 + 12d = 1r^2$ $1 + 12\left(\frac{r-1}{2}\right) = r^2$ $1 + 6r - 6 = r^2$ $r^2 - 6r + 5 = 0$ $(r-1)(r-5) = 0$ $r = 1$ or $r = 5$ 1; 5; 25 OR $r = 1 + 2d$ ⟶ $1 + 12d = r^2$ $1 + 12d = (1+2d)^2$ $= 1 + 4d + 4d^2$ $4d^2 - 8d = 0$ $4d(d-2) = 0$ $d = 0$ or $d = 2$ $r = 1 + 2(0)$ $r = 1 + 2(2)$ $= 1$ $= 5$ 1; 5; 25	✓ $a + 2d = ar$ ✓ $a + 12d = ar^2$ ✓ values of d ✓ standard form ✓ factors ✓ values of r ✓ sequence ✓ $a + 2d = ar$ ✓ $a + 12d = ar^2$ ✓ values of r ✓ standard form ✓ factors ✓ values of d ✓ sequence (6)

2.4.2	$S_n = \frac{a(r^n - 1)}{r - 1}$ $S_7 = \frac{1(5^7 - 1)}{5 - 1}$ $= 19531$	✓ substitute in correct formula ✓ answer (2)
		[27]

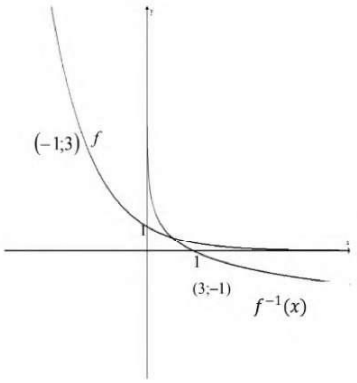
QUESTION 3

3.1	$y = 2^x$ $x = 0$ $\therefore f(0) = 2^0 = 1$ $\therefore C(0;1)$ y - intercept: $g(x) = -(x-1)^2 + q$ $C(0;1)$ $1 = -(0-1)^2 + q$ $1 = -1 + q$ $\therefore q = 2$	✓ substitute x = 0 and y = 0 into correct formula ✓ $x = -4$ ✓ $y = -2$ ✓ Symmetry E(4 ; 0) (4)
3.2	$g(x) = -(x-1)^2 + 2$ D(1; 2)	✓✓answer /*antwoord* (2)
3.3	$t = 2$	✓ answer/*antwoord* (1)
3.4	$f^{-1} : x = 2^y$ $\therefore y = \log_2 x$	✓✓ answer/*antwoord* (2)
3.5		✓ form of the graph /*vorm van die grafiek* ✓ x – intercept/*afsnit* ✓ any other coordinate on the graph /*enige ander koordinaat op die grafiek* (3)
3.6	$g(x) = -(x-1)^2 + 2$ $g(x+1) - 2 = -(x-1+1)^2 + 2 - 2$ $\therefore h(x) = -x^2$	✓ +1 and – 2 ✓ answer/*antwoord* (2)
3.7	Domain: $x \geq 0$ or $x \leq 0$	✓Answer (1)
		[08]

QUESTION 4

4		
4.1	$f(x)=\frac{2+x}{x-1}$ $f(x)=\frac{x-1+3}{x-1}$ $f(x)=\frac{x-1}{x-1}+\frac{3}{x-1}$ $f(x)=\frac{3}{x-1}+1$	✓ $\frac{x-1+3}{x-1}$ ✓ $\frac{x-1}{x-1}+\frac{3}{x-1}$ ✓ $f(x)=\frac{3}{x-1}+1$ (3)
4.2	$x=1$ $y=1$	✓ $x=1$ ✓ $y=1$(2)
4.3	$0=\frac{2+x}{x-1}$ $0=2+x$ $x=-2$ $A(-2;0)$	✓ $y=0$ ✓ $x=-2$ (2)
		[07]

QUESTION 5

5.1.1	$x=\left(\frac{1}{3}\right)^y$ $f^{-1}(x)=\log_{\frac{1}{3}} x$ OF $f^{-1}(x)=-\log_3 x$ Slegs Antwoord:Volpunte	✓ $x=\left(\frac{1}{3}\right)^y$ ✓✓antwoord **(3)**
5.1.2		$f(x)=\left(\frac{1}{3}\right)^x$: ✓ vorm ✓ y-afsnit $f^{-1}(x)=\log_{\frac{1}{3}} x$: ✓ vorm ✓ x-afsnit (4)
5.2	$p(-3)=10$ en $p'(x)=-2$ $p(x)=-2x+c$ $10=-2(-3)+c$ $\therefore c=16$ $p(x)=y=-2x+16$ $x=-2y+16$ $p^{-1}(x)=-\frac{1}{2}x+8$	✓ $m=-2$ ✓ $c=16$ ✓ omruil van x en y $(x=-2y+16)$ ✓ antwoord (4)
		[10]

QUESTION 6

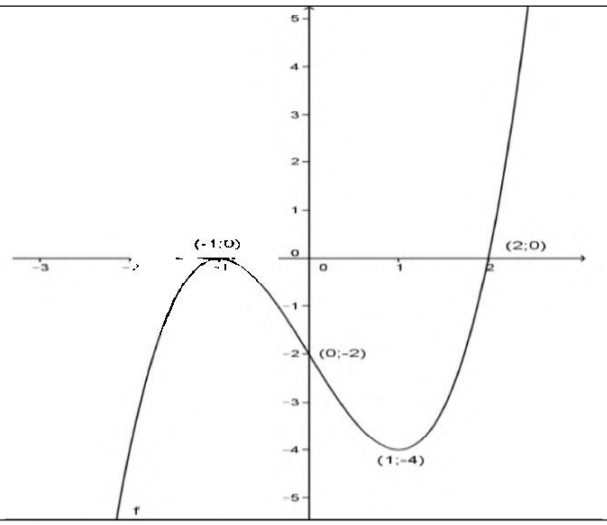
6.1	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $= \frac{2500\left[\left(1+\frac{0,06}{12}\right)^{60} - 1\right]}{\frac{0,06}{12}}$ $= \text{R}174\ 425,08$	✓ $n = 60$ and $i = \frac{0,06}{12} / 0,005$ ✓ correct substitution into correct formula ✓ answer (3)
6.2.1	After eleven months, Genevieve will owe/ *Na elf maande skuld Genevieve* $\text{A} = 82\,000\left(1+\frac{0,15}{12}\right)^{11}$ $= \text{R}\ 94\,006,79$	✓ $n = 11$ ✓ correct substitution into correct formula ✓ answer (3)
6.2.2	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $94\,006,79 = \frac{3\,200\left[1-\left(1+\frac{0,15}{12}\right)^{-n}\right]}{\frac{0,15}{12}}$ $\frac{94\,006,79}{3\,200} \times \frac{0,15}{12} = 1-\left(1+\frac{0,15}{12}\right)^{-n}$ $\left(1+\frac{0,15}{12}\right)^{-n} = 1-0,3672147...$ $-n\log\left(1+\frac{0,15}{12}\right) = \log 0,6327852...$ $-n = -36,8382...$ $n = 36,84$ Genevieve will have to pay 36 installments of R3 200	✓ 94006,79 ✓ substitute into correct formula ✓correct use of logs (logs to be defined) ✓ n =36,84 ✓ 36 installments (5)

6.2.3	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $= \frac{3200\left[1-\left(1+\frac{0,15}{12}\right)^{-0,83826912}\right]}{\frac{0,15}{12}}$ $P = 2652$ Outstanding balance after 36 installments is R2 652 Final payment will be: $A = 2652,00\left(1+\frac{0,15}{12}\right)^{1}$ $= R\ 2685,00$ **OR/*OF*** Balance : $94006,79\left(1+\frac{0,15}{12}\right)^{36} - \frac{3\,200\left[\left(1+\frac{0,15}{12}\right)^{36}-1\right]}{\frac{0,15}{12}}$ $= R2\ 651,72$ Final payment will be: $A = 2651,72\left(1+\frac{0,15}{12}\right)^{1}$ $= R\ 2\,685,00$	✓$n = -083826912$ ✓substitute into correct formula ✓answer ✓ $2652,00\left(1+\frac{0,15}{12}\right)^{1}$ ✓answer **OR/*OF*** ✓ $94006,79\left(1+\frac{0,15}{12}\right)^{36}$ ✓ $\frac{3\,200\left[\left(1+\frac{0,15}{12}\right)^{36}-1\right]}{\frac{0,15}{12}}$ ✓ 2 651,72 ✓ $2651,72\left(1+\frac{0,15}{12}\right)^{1}$ ✓answer (5)
		[16]

QUESTION 7

7.1	$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x) = \lim_{h\to 0}\frac{-2(x+h)^2-(-2x^2)}{h}$ $f'(x) = \lim_{h\to 0}\frac{-2x^2-4xh-2h^2+2x^2}{h}$ $f'(x) = \lim_{h\to 0}\frac{-2h(4x+h)}{h}$ $f'(x) = \lim_{h\to 0}\frac{-4x-h}{1}$ $f'(x) = -4x$	✓Substitution into correct formula ✓Expansion ✓Factoring ✓Applying limits ✓Answer
7.2.1	$y = \frac{2x^2}{x^{\frac{1}{2}}} - \frac{1}{x^{\frac{1}{2}}}$ $y = 2x^{\frac{3}{2}} - x^{-\frac{1}{2}}$ ✓ $\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$	✓ Simplified to binomial. ✓ $3x^{\frac{1}{2}}$ ✓$+\frac{1}{2}x^{-\frac{3}{2}}$ (3)
7.2.2	$D_x[9x^2 - 12x + 4]$ $= 18x - 12$	✓$9x^2 - 12x + 4$ ✓ $18x$ ✓-12 (3)
7.3	$\boldsymbol{y = x^{-2}}$ $\boldsymbol{\frac{dy}{dx} = -2x^{-3} = -\frac{2}{x^3}}$ Therefore $\frac{dy}{dx} < 0$ for $x > 0$	✓ $-2x^{-3}$ ✓$-\frac{2}{x^3}$ ✓ $\frac{dy}{dx} < 0$ for $x > 0$ (3)
		[14]

QUESTION 8

8.1		✓ $(-1;0)$ ✓ $(2;0)$ ✓ $(0;-2)$ ✓ $(1;-4)$ ✓ shape (5)
8.2	$-1 < x < 1$	(2)
8.3	Pt. of inflection: "halfway" between turning points(x-values) $\therefore x = \frac{-1+1}{2}$ $\therefore x = 0$	✓$x = \frac{-1+1}{2}$ ✓$x = 0$ (2)
8.4	$f''(x) > 0$ $\therefore x > 0$	✓$f''(x) > 0$ ✓$x > 0$ (2)
		[11]

QUESTION 9

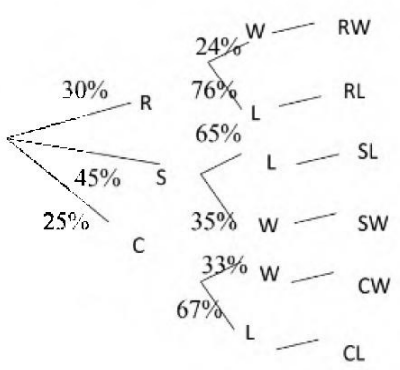
9.1	There are four horizontal bars and three vertical bars. $\Rightarrow 4y+3h=12$ $4y=12-3h$ $y=\frac{1}{4}(12-3h)$	✓ $4y+3h$ ✓ $4y+3h=12$ (2)
9.2	$Area = lb$ $= yh$ $=\frac{1}{4}(12-3h)h$ $=3h-\frac{3}{4}h^2$	✓ A=yh ✓ Subst of y ✓ simplification (3)
9.3	$Area = 3h-\frac{3}{4}h^2$ $\frac{dA}{dh}=3-\frac{3}{2}h$ $\frac{dA}{dh}=0$ $3-\frac{3}{2}h=0$ $h=2$ $y=\frac{1}{4}(12-3(2))$ $=\frac{3}{2}$	✓ derivative ✓ equating to 0 ✓ h =2 ✓ subst of h ✓ value of y (5)
		[10]

QUESTION10

10.1	$\frac{10!}{2.2.2} = 453600$	✓$\frac{10!}{2!.2!.2!}$ ✓Answer (2)
10.2	$\frac{8!}{4} = 10080$	✓$\frac{8!}{4}$ ✓Answer (2)
10.3	Same letters are together in 2.2.2.7! ways = Probability $= \frac{8.7!}{\frac{10!}{8}} = \frac{4}{45}$ So required probability is $\frac{41}{45}$ OR Same letters not together is 41380 ways Probability required is $\frac{41}{45}$	✓ Same letters together ✓ Probability of same letters together ✓ Answer OR ✓Same letter together ✓Same letter not together ✓Answer (3)
		[07]

QUESTION 11

11.1.1	$P(\text{A or B}) = P(A) + P(B)$ $0.88 = 0.5 + x$ $x = 0.38$	✓ subst into correct formula ✓ answer (2)
11.1.2	$P(\text{A or B}) = P(A) + P(B) - P(A).P(B)$ $0.88 = 0.5 + x - 0.5x$ $0.38 = 0.5x$ $x = 0,76$	✓ subst into correct formula ✓ $0.5x$ ✓ answer (3)

11.2.1		✓ 25% for C ✓ Correct outcomes to represent win/lose ✓ Correct diagram (3)
11.2.2	Probability of winning P(W) = P(R and W) or P(S and L) or P(C and W) $=30\% \times 24\% + 45\% \times 65\% + 25\% \times 67\%$ $= 0.9445 \text{ OR } \frac{1889}{2000}$	✓ $30\% \times 24\% + 45\% \times 65\% + 25\% \times 67\%$ ✓ . ✓ answer (3)
		[11]