LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

NATIONAL SENIOR CERTIFICATE

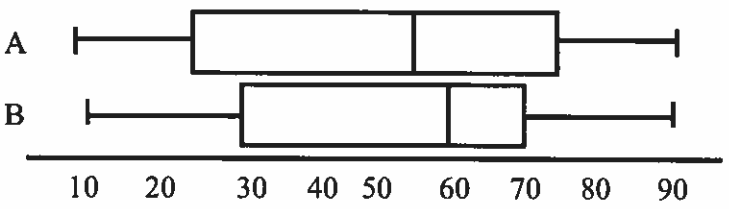
GRADE/GRAAD 12

MATHEMATICS PAPER 2/ WISKUNDE V2
SEPTEMBER 2020
MEMORANDUM

MARKS/PUNTE: 150

This memorandum consists of 15 pages / Hierdie memo bestaan uit 15 bladsye.

QUESTION /*VRAAG* 1

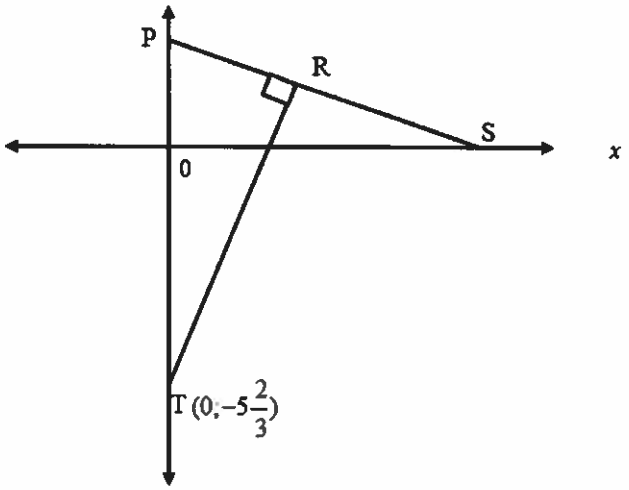
1.1.1		✓box /*mond* ✓whiskers, on scale/ *stert, en op skaal* (2)
1.1.2	School B/*skool B*: In B 50% of learners got above 60% whilst 50% of learners of A above 55% / 50% *van B het meer as* 60% ; *by A het* 50% *meer as* 55% median of B > median of A/ *mediaan van B > mediaan van A* 75% of B above 30% and at A 75% above 23,5% / 75% *van B bo* 30% ; *By A* 75% *bo* 23,5% OR/OF Skewed to the right/ *skeef na Regs*	✓School /*skool B* ✓✓any two reasons *enige twee redes* ✓✓ OR/ OF ✓✓ (3)
1.2	$\frac{135\times 24+225\times 32+200\times x}{560}=x$ $3240+7200+200x=560x$ $360x=10440$ $x=29$	✓numerator/ *teller* ✓560 ✓$360x=10440$ ✓ answer/*antwoord* (4)
		[9]

QUESTION/*VRAAG* 2

2.1	$a=23{,}19\ \ b=0{,}768..$ $y=23{,}19+0{,}77x$	✓a ✓b ✓ $y=23{,}19+0{,}77x$ (3)
2.2	$y=23{,}19+{,}77(30)$ $=46{,}29$ $\approx 46\%$	✓ substitution/*vervanging* ✓answer/*antwoord* (2)

2.3.1	$\delta = 10{,}99$	✓✓answer /*antwoord* (2)
2.3.2	$57{,}4\% - 49{,}4\% = 8\%$ $\frac{x}{50} = \frac{8}{100}$ $x = 4$ marks	✓8% ✓4 marks/ *punte* (2)
		[9]

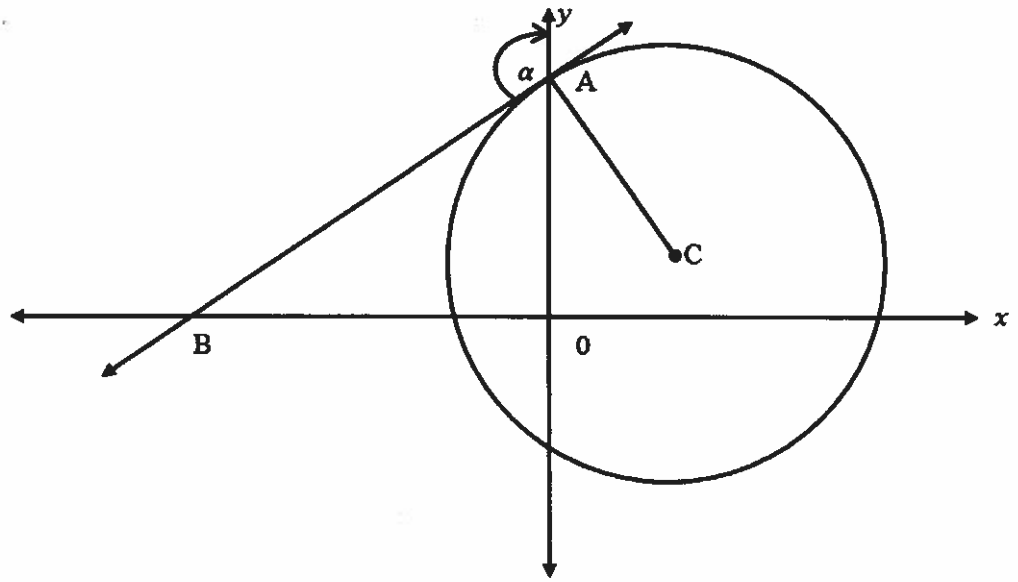
QUESTION /*VRAAG* 3



3.1	$x + ay = a$ $o + ay = a$ $y = 1$ $\therefore$ P(0;1)	✓x=0 ✓y=1 ✓ P(0;1) (3)
3.2	$OS : OP = 3 : 1$ $S(3;0) : x + ay = a$ $3 + a(0) = a$ $a = 3$	✓ $S(3;0)$ ✓substitution/*vervang* ✓ $a = 3$ (3)
3.3	$m_{PS} = -\frac{1}{3}$ $m_{TR} = 3$ RT: $y = 3x - 5\frac{2}{3}$	✓ $m_{PS} = -\frac{1}{3}$ ✓ $m_{TR} = 3$ ✓equation/ *vergelyking* (3)
3.4	$x + 3y = 3 \Rightarrow y = -\frac{1}{3}x + 1$ $\therefore -\frac{1}{3}x + 1 = 3x - \frac{17}{3}$ $-x + 3 = 9x - 17$ $20 = 10x$	✓equating /*stel gelyk*

	Substitute: $y=-\frac{1}{3}(2)+1$ $y=\frac{1}{3}$ $R(2;\frac{1}{3})$	✓ simplification/*vereenvoudig* ✓ $x=2$ ✓ $y=\frac{1}{3}$ (4)
3.5	$area\Delta PRT=\frac{1}{2}\times PT\times h$ $=\frac{1}{2}\times 6\frac{2}{3}\times 2$ $=\frac{20}{3}$ sqd units/ *vierkante eenhede*	✓ $PT=6\frac{2}{3}$ ✓ $h=2$ ✓answer/ *antwoord* (3)
3.6	PT is the diameter(converse: ∠ in semi ⊙)/ ⊙*PT is die middellyn (omgekeerde: ∠ in semi* ⊙) $radius=\frac{10}{3}$ units.	✓S/R ✓answer / *antwoord* (2)
		[18]

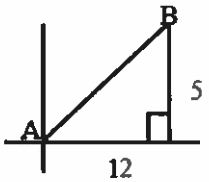
QUESTION/*VRAAG* 4



4.1.1	$x^2-6x+9+y^2-4y+4=12+9+4$ $(x-3)^2+(y-2)^2=25$ $C(3;2)$ $r=5$	Completing the square *vierkantsvoltooing* ✓LHS/ *LK* ✓RHS/*RK* ✓ $C(3;2)$ ✓ $r=5$ (4)
4.1.2	Y-intercept: /*Y as*: $x=0$ $y^2-4y-12=0$ $(y-6)(y+2)=0$ $y=-2$ or $y=6 \therefore A(0;6)$ $m_{AC}=\frac{6-2}{0-3}=-\frac{4}{3}$ $m_{AB}=\frac{3}{4}$ (tan ⊥ radius) $y=\frac{3}{4}x+6$	✓ $x=0$ ✓standardform/ *standaardvorm* ✓factors/*faktore* ✓ $A(0;6)$ ✓ $m_{AC}=-\frac{4}{3}$ ✓ $m_{AB}=\frac{3}{4}$ ✓R ✓equation/ *vergelyking* (8)
4.1.3	*Let* $A=\theta$: $\tan\theta=\frac{3}{4}$ $\theta=36{,}87°$ $\alpha=90°+36{,}87°$ $\alpha=126{,}87°$	✓ $\tan\theta=\frac{3}{4}$ ✓ $\theta=36{,}87°$ ✓ $\alpha=90°+36{,}87°$ ✓answer / *antwoord* (4)

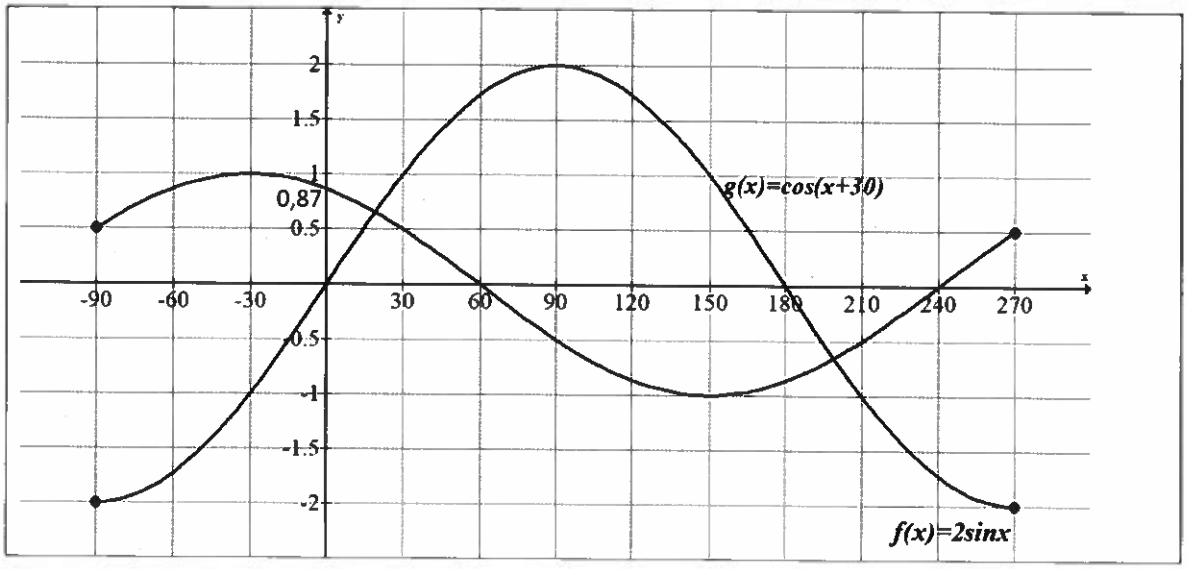
4.2	$x^2+2x+1+y^2-4y+4=44+1+4$ $(x+1)^2+(y-2)^2=49$ Centre / *middelpunt* D(−1;2) ; radius = 7 CD = 3 − −1 = 4 $r_C+r_D=5+7=12>4$ ∴ the two circles intercept internally/ *twee sirkels sny inwendig*	✓Centre D(−1;2) ✓r = 7 ✓CD = 4 ✓$r_C+r_D=12$ ✓>4 ✓conclusion / *gevolgtrekking* (6)
		[22]

QUESTION/*VRAAG* 5

5.1	$\tan A=\frac{5}{12}$  $r^2=12^2+5^2$ $=169$ $r=13$ $13\sin A-3\tan B$ $=13(\frac{5}{13})-3(\frac{12}{5})$ $=5-\frac{36}{5}$ $=-\frac{11}{5}$	✓diagram ✓r = 13 ✓$\sin A=\frac{5}{13}$ ✓$\tan B=\frac{12}{5}$ ✓answer /*antwoord* (5)
5.2	$\frac{-\tan 60°-\sin x}{\sin x+2\cos 30°}$ $=\frac{-\sqrt{3}-\sin x}{\sin x+2(\frac{\sqrt{3}}{2})}$ $=\frac{-(\sin x+\sqrt{3})}{(\sin x+\sqrt{3})}$ $=-1$	✓$-\tan 60°$ ✓$-\sin x$ ✓$2\cos 30°$ ✓special angles/ *bekende hoeke* ✓factorisation/ *faktoriseer* ✓answer/

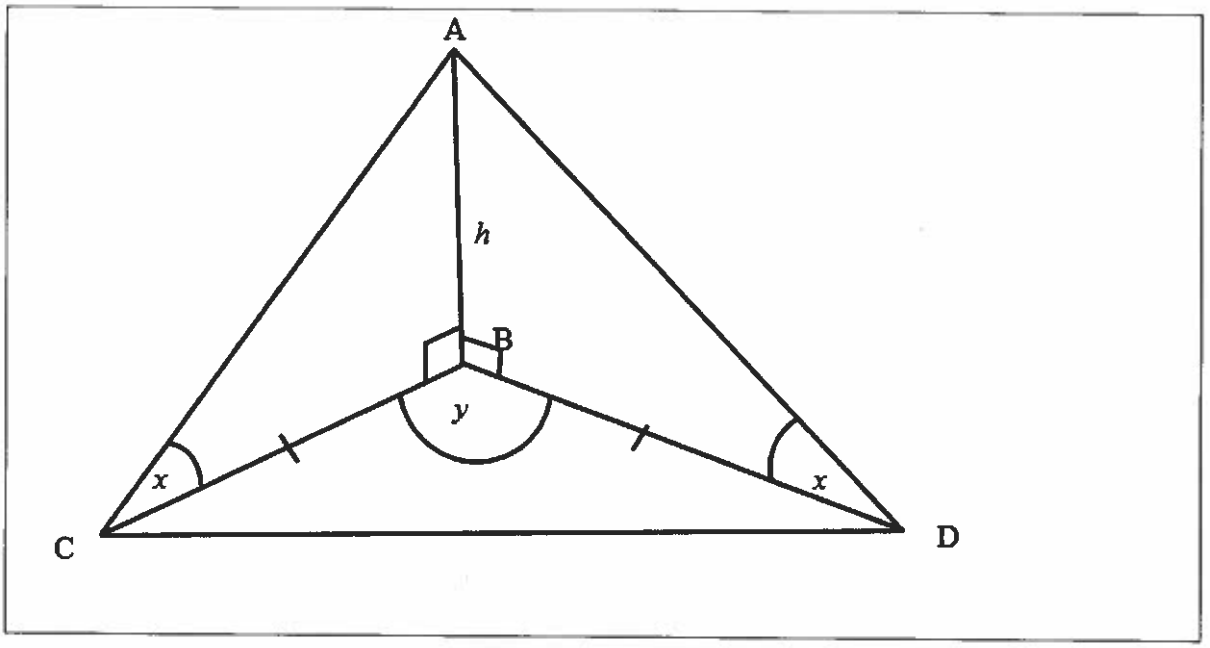
5.3.1	$\cos(x+y)=\sin(90°-(x+y)$ $=\sin((90°-x)+(-y))$ $=\sin(90°-x)\cos(-y)+\cos(90°-x)\sin(-y)$ $=\cos x\cos y-\sin x\sin y$	✓Correct co-fn/ *korrekte ko – funksie* ✓ regrouping/ *herrangskik* ✓Correct expansion/ *uitbreiding* ✓simplification/ *vereenvoudig* (4)
5.3.2	$\cos(x-y)-\cos(x+y)$ $=\cos x\cos y+\sin x\sin y-(\cos x\cos y-\sin x\sin y)$ $=\cos x\cos y+\sin x\sin y-\cos x\cos y+\sin x\sin y$ $=2\sin x\sin y$	✓Expansion/ *brei uit* ✓simpilification / *vereenvoudig* (2)
5.3.3	$2\sin 195°\sin 45°$ $=\cos(195°-45°)-\cos(195°+45°)$ $=\cos 150°-\cos 240°$ $=-\cos 30°-(-\cos 60°)$ $=-\frac{\sqrt{3}}{2}+\frac{1}{2}$ $=\frac{-\sqrt{3}+1}{2}$	✓Substitution/ *vervang* ✓$-\cos 30°$ ✓$-\cos 60°$ ✓Special angles/ *bekende hoeke* ✓answer/*antwoord* (5)
		[22]

QUESTION/*VRAAG* 6



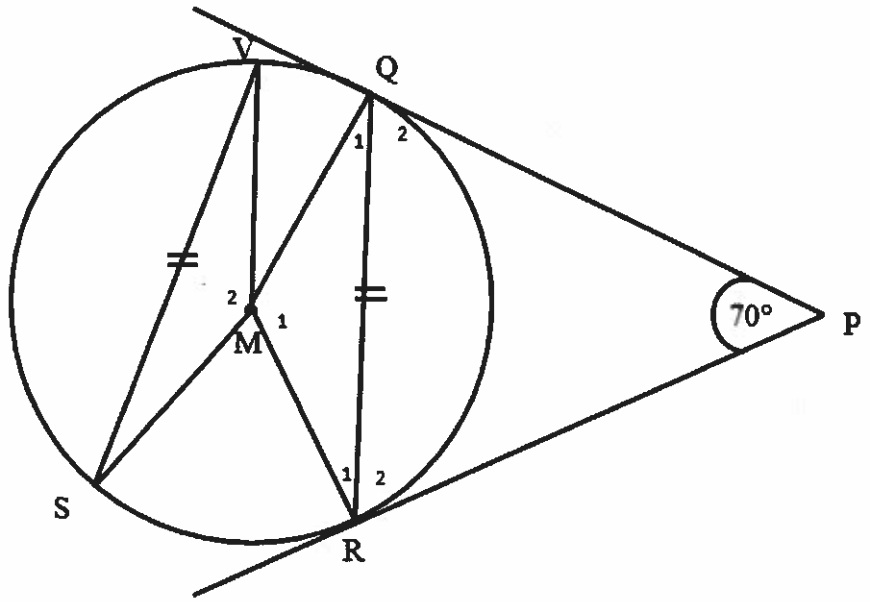
6.1	*g*: ✓x-intercepts/ $x-asse$ ✓y-intercepts/$y-as$ ✓ turningpoints / *draaipunte*	(3)
6.2	$2\sin x = \cos x\cos 30° - \sin x\sin 30°$ $2\sin x = \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$ $\frac{5}{2}\sin x = \frac{\sqrt{3}}{2}\cos x$ $\frac{\sin x}{\cos x} = \frac{\frac{\sqrt{3}}{2}}{\frac{5}{2}}$ $\tan x = \frac{\sqrt{3}}{5}$	✓Expansion of *g*/ *brei uit* ✓Special angles/ *bekende hoeke* ✓$\frac{5}{2}\sin x = \frac{\sqrt{3}}{2}\cos x$ ✓$\frac{\sin x}{\cos x}$ (4)
6.3	$\tan x = \frac{\sqrt{3}}{5}$ $x = 19{,}1°$ *or* $x = 180° + 19{,}1° = 199{,}1°$	✓19,1° ✓199,1° (2)
6.4.1	$x \in [60°;180°] \cup [240°;270°]$	✓60°*and* / *en*180° ✓240°*and* / *en*270° ✓notation / *notasie* (3)
6.4.2	$x \in [19{,}1°;199{,}1°]$	✓criticalvalues / *kritieke waardes* ✓notation / *notasie* (2)

QUESTION /*VRAAG 7*



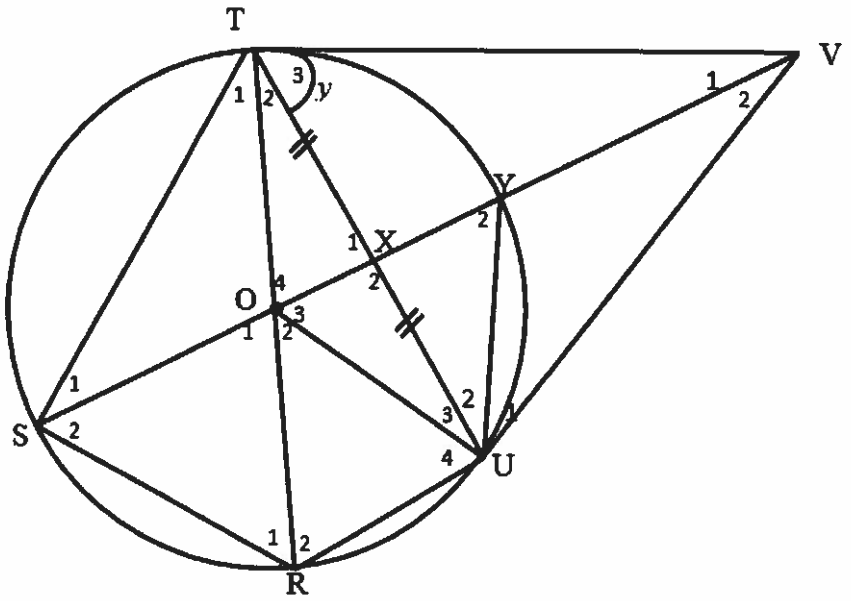
	$In\,\Delta ABC: \tan x = \frac{h}{CB}$	✓ $\tan x = \frac{h}{CB}$
	$CB = \frac{h}{\tan x}$	✓ $CB = \frac{h}{\tan x}$
	$In\,\Delta BCD: CB = BD$ $CD^2 = BC^2 + BD^2 - 2BC \times BD \times \cos y$	✓Correct application of cos rule/ *korrekte gebruik van cosreël*
	$= (\frac{h}{\tan x})^2 + (\frac{h}{\tan x})^2 - 2\frac{h}{\tan x} \times \frac{h}{\tan x} \times \cos y$	✓substitution
	$= \frac{2h^2}{\tan^2 x} - \frac{2h^2 \cos y}{\tan^2 x}$ $= \frac{2h^2}{\tan^2 x}(1 - \cos y)$	✓ simplify/ *vereenvoudig*
	$CD = \frac{h}{\tan x}\sqrt{2(1-\cos y)}$ $CD = \frac{h}{\tan x}\sqrt{2-2\cos y}$	✓factorising/ *faktoriseer*
		[6]

QUESTION/*VRAAG* 8



8.1.1	$\hat{Q}_2 = \hat{R}_2$ (tangents from same point)/ *raaklyne uit dsde punt*) $2\hat{R}_2 + 70° = 180°$ (*∠'sof ΔPQR/ ∠'e van ΔPQR*) $2\hat{R}_2 = 110°$ $\hat{R}_2 = 55°$	✓S✓R ✓S ✓answer/ *antwoord* (4)
8.1.2	$\hat{Q}_1 + \hat{Q}_2 = 90°$ (tangent ⊥ *radius*)/(*raaklyn ⊥ op radius*) $\hat{Q}_1 + 55° = 90°$ $\hat{Q}_1 = 35°$	✓S✓R ✓answer / *antwoord* (3)
8.1.3	$\hat{Q}_1 = \hat{R}_1 = 35°$ (r =) $\hat{M}_1 = 110$ (*∠'sof ΔMQR*) $\hat{M}_2 = \hat{M}_1 = 110$ (= chords subtend =∠'s at centre)/ (= *koorde = middelpunts ∠'e*)	✓S/R ✓ $\hat{M}_2 = 110$ ✓R (3)

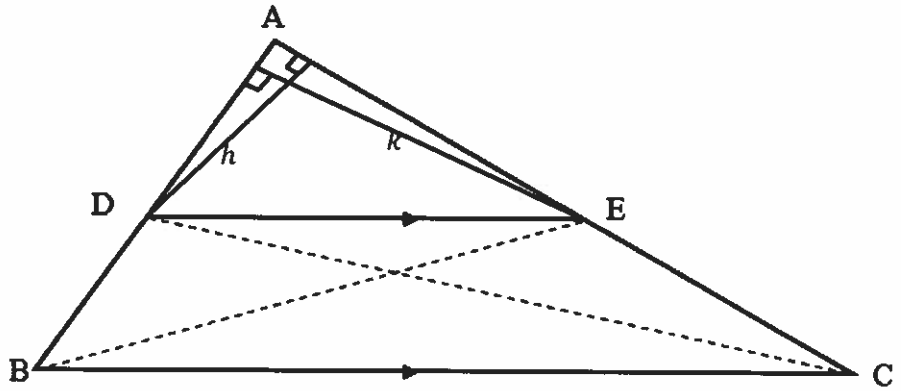
8.2



8.2.1	$\hat{X}_1 = 90°$ (line from centre to midpoint of chord)/ *(van midpunt van ⊙, mdpt van koord)* $\hat{U}_3 + \hat{U}_4 = 90°$ (∠ in semi ⊙) $\therefore \hat{X}_1 = \hat{U}_3 + \hat{U}_4 = 90°$ $\Rightarrow RU \parallel SV$ (corresponding ∠'s =)/ (ooreenkomstige ∠'e =)	✓S ✓R ✓S ✓S ✓R (5)
8.2.2	$\hat{R}_2 = y$ (tan-chord theorem)/ *(∠ tussen raaklyn en koord)* $\hat{O}_1 = y$ (alternate ∠'s, $RU \parallel SV$)/ verwisselende ∠'e, $RU \parallel SV$) $\hat{O}_1 = 2\hat{T}_1$ *(∠ at centre = 2∠at circumference)* *(middelpuntsꝺ = 2×omtreksꝺ)* $\therefore \hat{T}_1 = \frac{1}{2}y$	✓S✓R ✓S ✓S✓R (5)
		[20]

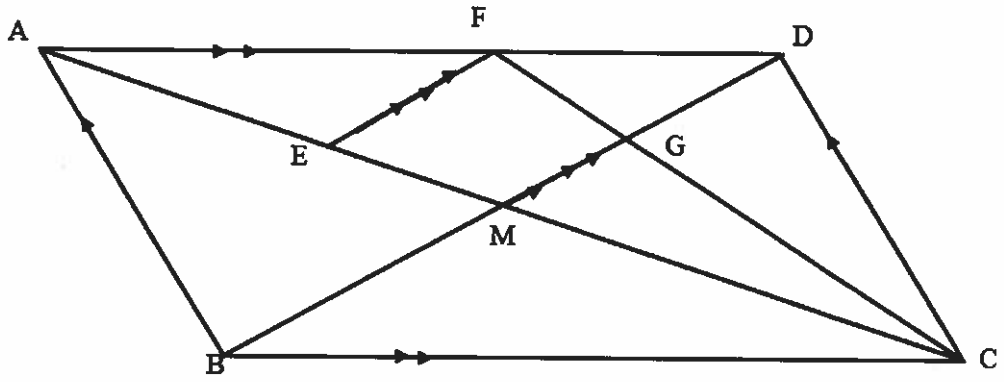
QUESTION/*VRAAG* 9

9.1



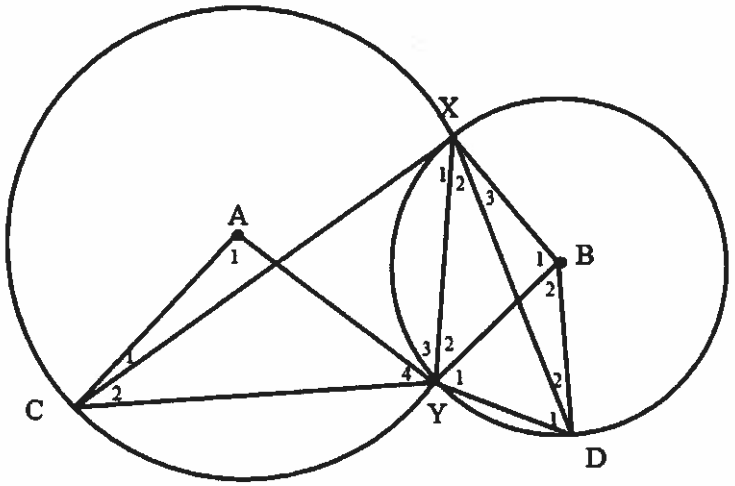
9.1	Construction: Draw altitudes h and k. *Konstruksie: trek hoogtelyne h en k.* $\frac{area\Delta ADE}{area\Delta BDE} = \frac{\frac{1}{2} \times AD \times h}{\frac{1}{2} \times BD \times h} = \frac{AD}{DB}$ $\frac{area\Delta ADE}{area\Delta DEC} = \frac{\frac{1}{2} \times AE \times k}{\frac{1}{2} \times EC \times k} = \frac{AE}{EC}$ But the $area\Delta BDE = area\Delta DEC$ (same base and same height/ *dsde basis en hoogte*) $\therefore \frac{area\Delta ADE}{area\Delta BDE} = \frac{area\Delta ADE}{area\Delta DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	✓Construction/ *konstruksie* ✓S ✓S ✓S✓R ✓S (6)

9.2



9.2.1	$\frac{AE}{EM}=\frac{AF}{FD}=\frac{4}{3}$ (line ‖to one side of ΔAMD)/ (*lyn ‖ aan een sy van ΔAMD*) $\frac{EM}{AM}=\frac{3}{7}$	✓S ✓R ✓answer (3)
9.2.2	$AM:MC=7:7$ (diagonals of parm) $\frac{CM}{ME}=\frac{7}{3}$	✓S✓R ✓answer (3)
9.2.3	$area\Delta BDC=area\Delta ABD$ (diagonals of parm) $\frac{\text{area}\Delta FDC}{\text{area}\Delta BDC}=\frac{area\Delta FDC}{area\Delta ABD}$ $=\frac{\frac{1}{2}\times FD\times height}{\frac{1}{2}\times AD\times height}$ $=\frac{FD}{AD}$ (between same parallel lines)/ (*tussen dieselfde ‖ lyne*) $=\frac{3}{7}$	✓S ✓replacing/ *vervang* ✓ area formula / *oppervlak formule* ✓R (4)
		[16]

QUESTION /*VRAAG* 10



10.1	*In* ΔXYC *and* ΔDYX : $\hat{X}_1 = \hat{D}_1$ (tan-chord theorem/ *∠ tussen raaklyn en koord*) $\hat{C}_2 = \hat{X}_2$ (tan-chord theorem/ *∠ tussen raaklyn en koord* $\hat{Y}_3 + \hat{Y}_4 = \hat{Y}_1 + \hat{Y}_2 (\angle' sof \Delta)$ $\therefore \Delta XYC \mid\mid\mid \Delta DYX$ $(\angle\angle\angle)$ $\Rightarrow \frac{XY}{DY} = \frac{YC}{YX}$ $\therefore XY^2 = YC.DY$	✓S✓ R ✓S ✓S ✓S (5)
10.2	$\hat{A}_1 = 2\hat{X}_1$ (∠ at centre is twice ∠ at circumference) (*middelpuntsꝺ = 2 × omtreksꝺ*) Similarly/*netso* $\hat{B}_1 = 2\hat{D}_1$ But $\hat{X}_1 = \hat{D}_1$ (tan-chord theorem/ *∠ tussen raaklyn en koord*) $\therefore \hat{A}_1 = \hat{B}_1$	✓S✓R ✓ ✓S (4)
10.3	$\frac{CA}{YB} = \frac{CY}{YX}$ (ΔCAY $\mid\mid\mid$ ΔYBX) $\frac{R}{r} = \frac{CY}{YX}$ $\therefore \frac{R^2}{r^2} = \frac{CY^2}{YX^2}$ $= \frac{CY.CY}{CY.DY}$ $= \frac{CY}{\quad}$	✓S ✓Substitute/ *vervang* ✓Square both sides/ *kwadreer albei kante*

	$\therefore \frac{r^2}{R^2} = \frac{DY}{CY}$	✓Replace/*vervang* YX^2 ✓answer /*antwoord* (5)
		[14]