GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

2015

10612

MATHEMATICS

SECOND PAPER

MARKS: 150
TIME: 3 hours

14 pages, 5 diagram sheets
and 1 information sheet.

MATHEMATICS: Paper 2
1061E
10612E

X05

P.T.O.

GAUTENG DEPARTMENT OF EDUCATION

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(Second Paper)

TIME: 3 hours

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INSTRUCTIONS AND INFORMATION

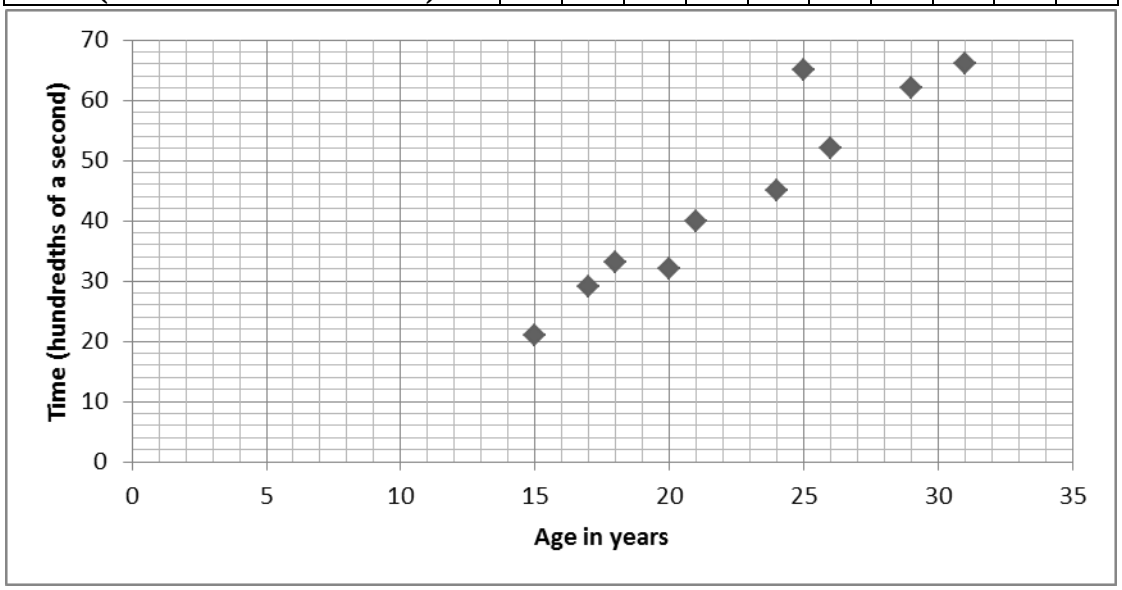
Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Five diagram sheets for QUESTION 1.1, 2.1, 2.2, 2.3, 9.1, 9.2, 10.1, 10.2, 11 and the INFORMATION SHEET are attached at the end of this question paper.
9. Write your name in the space provided on the Diagram Sheets and hand them in together with your ANSWER BOOK.
10. Number the answers correctly according to the numbering system used in this question paper.
11. Write neatly and legibly.

QUESTION 1

A hospital carries out a survey to compare the reaction time of patients of different ages, to a specific medication taken.
The results are shown in the table and scatter plot below.

Age in years	15	17	18	20	21	24	25	26	29	31
Time (hundredths of a second)	21	29	33	32	40	45	65	52	62	66



1.1 Draw the line of best fit on the scatter plot on DIAGRAM SHEET 1. (2)

1.2 ONE of the patient's reaction time is an outlier.

1.2.1 How old is this patient? (1)

1.2.2 Explain why this patient is an outlier. (1)

1.3 Calculate the equation of the least squares regression line (line of best fit) for the data. (3)

1.4 Calculate the correlation coefficient of the data. Comment on the strength of the relationship between the variables. (2)

1.5 Hospital records for this reaction time test give the following information.

	15 year olds	30 year olds
	Time (hundredths of a second)	Time (hundredths of a second)
Lower quartile	20	61
Median	22	65
Upper quartile	25	76

Comment on the reaction time of the different age groups on this test. Motivate your answer by referring to the values given. (2)

[11]

QUESTION 2

The ages of 500 people attending a concert are given in the table below.

Age in years	Number of people	Cumulative frequency
$0 \leq A < 10$	20	
$10 \leq A < 20$	130	
$20 \leq A < 30$	152	
$30 \leq A < 40$	92	
$40 \leq A < 60$	86	
$60 \leq A < 80$	18	
$80 \leq A < 100$	2	

2.1 Complete the cumulative frequency column on DIAGRAM SHEET 2. (1)

2.2 Draw an ogive (cumulative frequency graph) of the above data on DIAGRAM SHEET 2. (3)

2.3 Use your cumulative frequency graph to estimate

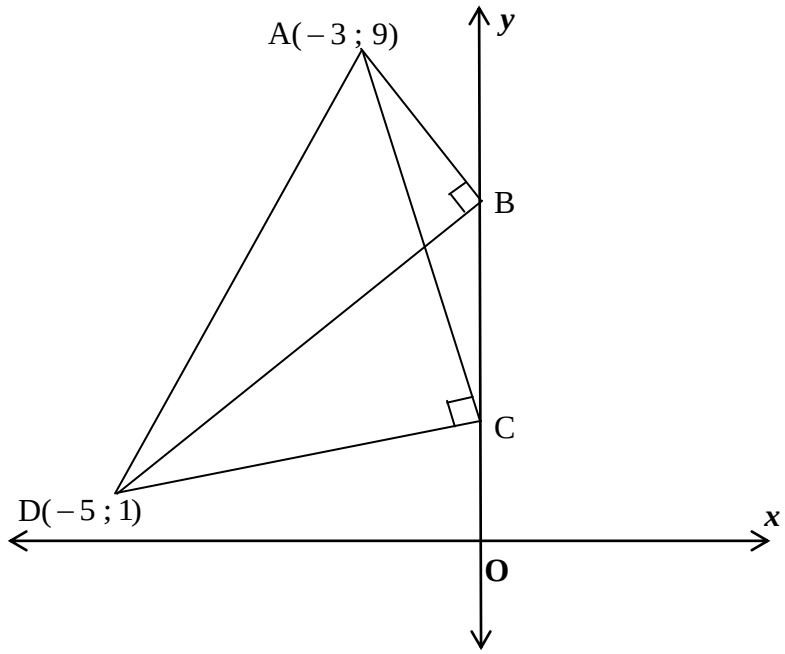
2.3.1 the median age. (1)

2.3.2 the percentage of people at the concert who are 16 years and older. (3)
[8]

QUESTION 3

In the diagram, $A(-3\ ;\ 9)$ and $D(-5\ ;\ 1)$ are points on ΔABD and ΔACD.

B and C are points on the *y*-axis such that $A\hat{B}D = A\hat{C}D = 90^\circ$



3.1 Calculate the coordinates of M, the midpoint of AD. (2)

3.2 Calculate the length of the radius of the circle passing through A, B and D. (2)

3.3 Will point C lie on circle ABD? Give a reason for your answer. (2)

3.4 Calculate the coordinates of B. (5)

3.5 Determine the equation of the straight line passing through D and which is parallel to AB. (3)

3.6 Calculate the size of $B\hat{D}A$. Round off the answer to the nearest degree. (6)

[20]

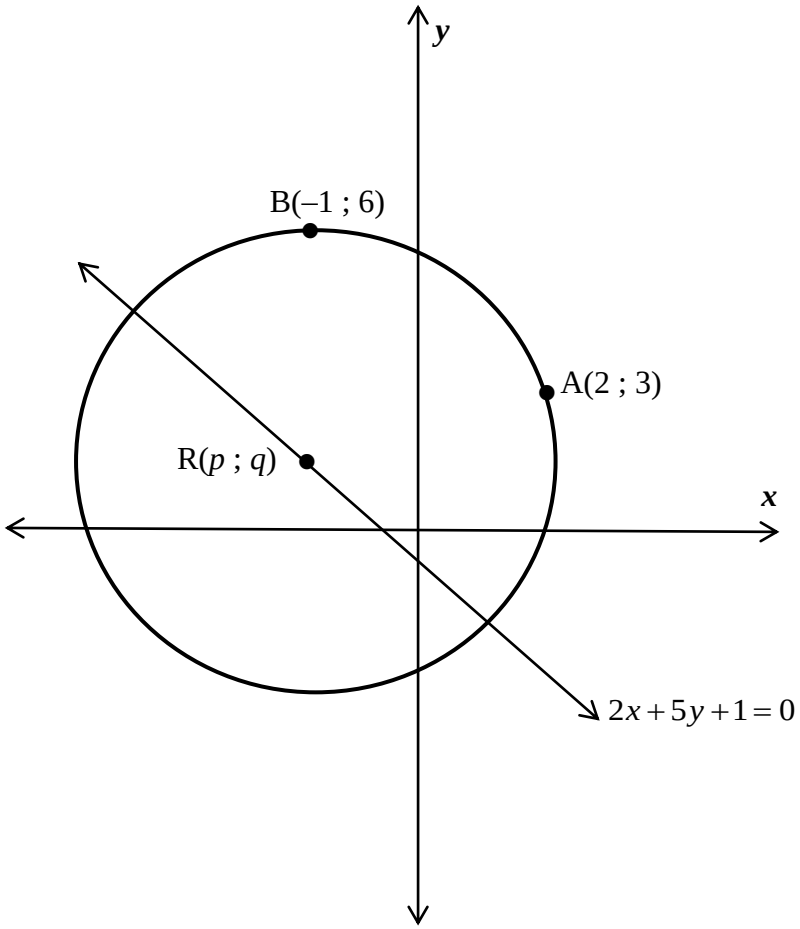
QUESTION 4

4.1 Given: $x^2 + y^2 - 2x + 6y = 0$.

4.1.1 Determine the coordinates of the centre of the circle and the length of the of the circle. (4)

4.1.2 Determine the equation of the tangent to the circle at $(-2 ; -4)$. (4)

4.2 Points A(2 ; 3) and B(–1 ; 6) lie on the circumference of the given circle.
R(p ; q) is the centre of the circle and lies on the line $2x + 5y + 1 = 0$.



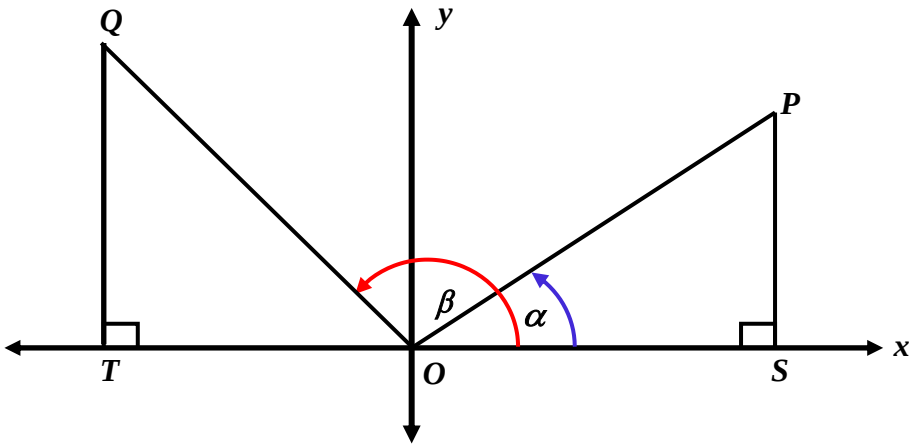
4.2.1 Show that $p - q = -4$. (4)

4.2.2 Hence, determine the equation of the circle. (7)
[19]

QUESTION 5

This question is to be done without the use of a calculator.

In the diagram below the equation of OP is given by $3y - 2x = 0$. S is a point on the x-axis such that PS $\perp$ x-axis. $S\hat{O}P = a$. The line segment OQ is drawn such that $S\hat{O}Q = \beta$. T is a point on the x-axis such that QT $\perp$ x-axis.



5.1 Show that $\tan\alpha = \frac{2}{3}$. (2)

5.2 Calculate the value of $\sin\alpha$. (2)

5.3 Write down $Q\hat{O}P$ in terms of α and β. (1)

5.4 If it is given that $\sin\beta = \frac{3}{5}$, find the value of $\sin Q\hat{O}P$. (4)

[9]

QUESTION 6

6.1 Simplify fully WITHOUT the use of a calculator:

$$\frac{\cos(40° - x)\cdot\cos x - \sin(40° - x)\cdot\sin x}{\sin 205°.\cos 25°} \quad (5)$$

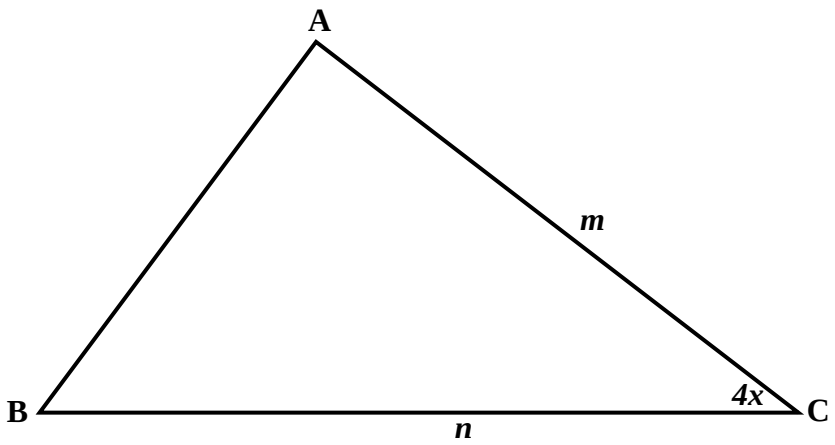
6.2 Given: $\frac{\cos 2x}{\cos x + \sin x}$

6.2.1 Show that: $\frac{\cos 2x}{\cos x + \sin x} = \cos x - \sin x.$ (2)

6.2.2 Show that $\cos x\left(\frac{\cos 2x}{\cos x + \sin x}\right) = \frac{1}{2}$ can be simplified to $\cos 2x = \sin 2x.$ (4)

6.2.3 Hence, determine the general solution of $\cos x\left(\frac{\cos 2x}{\cos x + \sin x}\right) = \frac{1}{2}$. (3)

6.3 In ΔABC, $AC = m$, $BC = n$ and $\hat{C} = x$.



6.3.1 Write down an expression for the area of ΔABC (1)

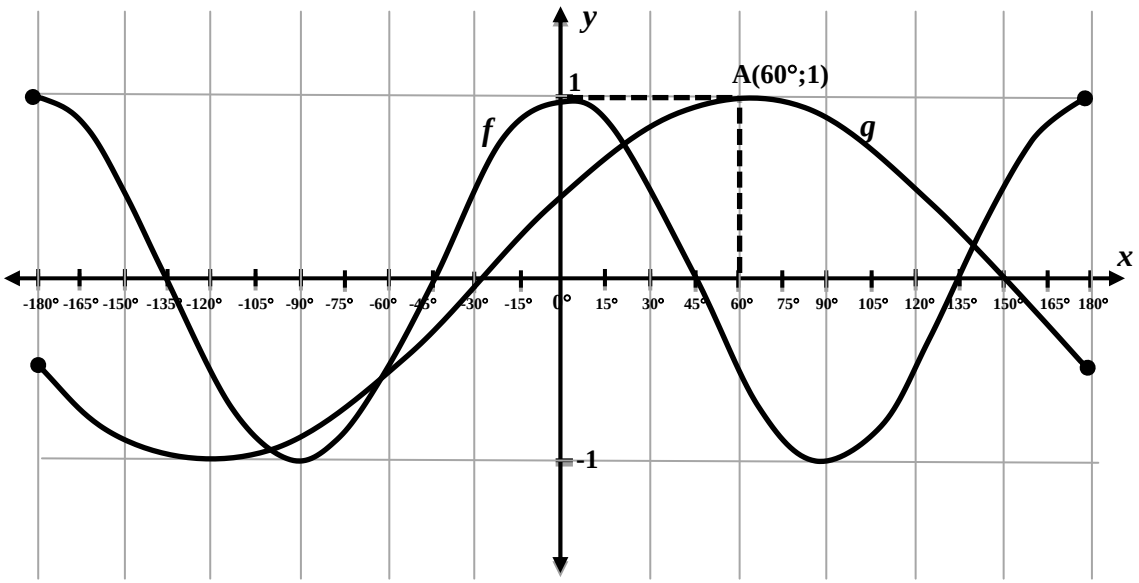
6.3.2 Calculate the value of x for which the area of the triangle will be a maximum. (2)

6.3.2 What conclusion can you make about the type of triangle formed when the area of the triangle is a maximum? (1)

[18]

QUESTION 7

The sketch below shows the graphs of $f(x) = \cos 2x$ and $g(x) = \sin(x - \theta)$ for $x \in [-180° ; 180°]$. $A(60° ; 1)$ is a point on the graph of g. Use the graph to answer the questions that follow.



7.1 Write down the value of θ. (1)

7.2 Determine the period of f. (1)

7.3 If $h(x) = f(x) - 1$, write down the range of h. (1)

7.4 Determine the values of x, where $x \in [0° ; 180°]$ for which

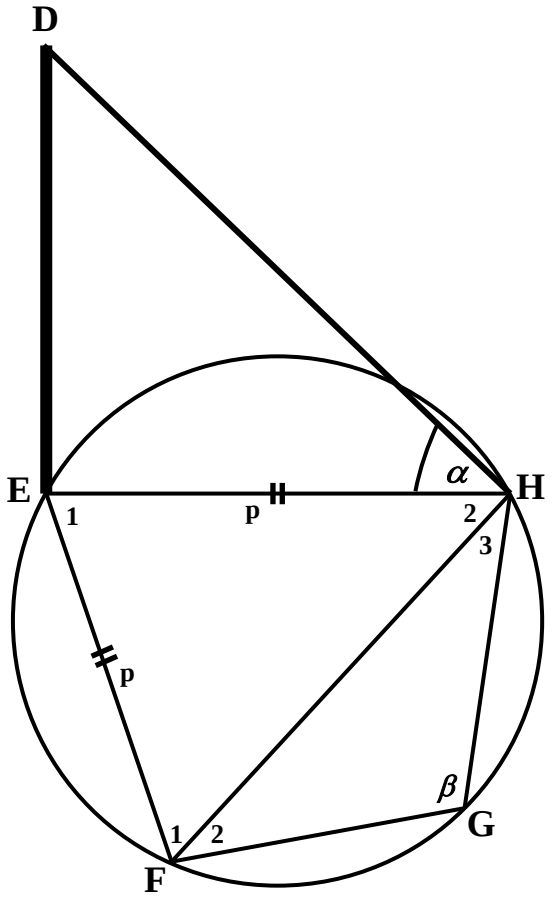
7.4.1 $f(x).g(x) < 0$ (3)

7.4.2 $f'(x).g(x) > 0$ (2)

[8]

QUESTION 8

In the diagram below, DE represents a vertical cell phone tower positioned on one corner of a field. The field is shaped as a cyclic quadrilateral EFGH and E, F, G and H are all on the same horizontal plane.. From H, the angle of elevation to D, the top of the cell phone tower, is α. EH = EF = p units. $\hat{G} = \beta$.



8.1 Write down DE in terms of α. (1)

8.2 Show that:

8.2.1 $\hat{H}_2 = \frac{1}{2}\beta$ (Give reasons for your answers). (3)

8.2.2 $p = \frac{FH}{2\cos\frac{1}{2}\beta}$ (3)

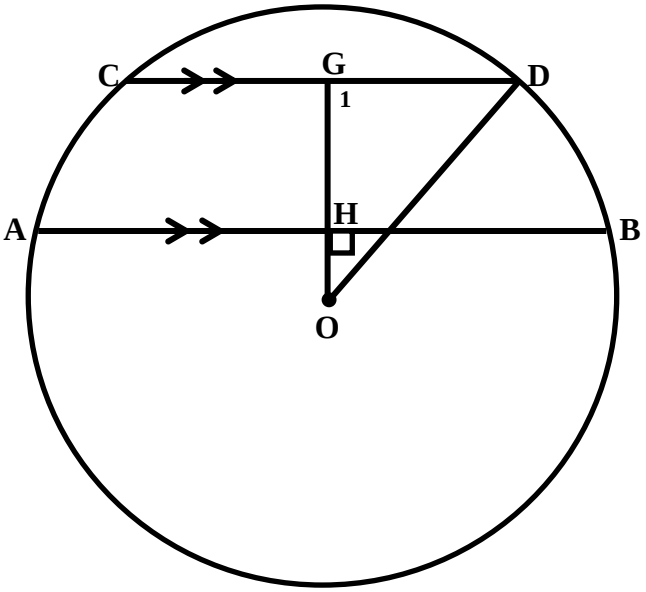
8.2.3 $FH = p\sqrt{2(1+\cos\beta)}$ (3)

[10]

GIVE REASONS FOR YOUR STATEMENTS IN QUESTIONS 9, 10, AND 11.

QUESTION 9

9.1 In the diagram, O is the centre of the circle with AB || CD and OH ⊥ AB, AB = 24 cm, CD = 10 cm and OD = 13 cm.



9.1.1 Give reasons for the statements below.

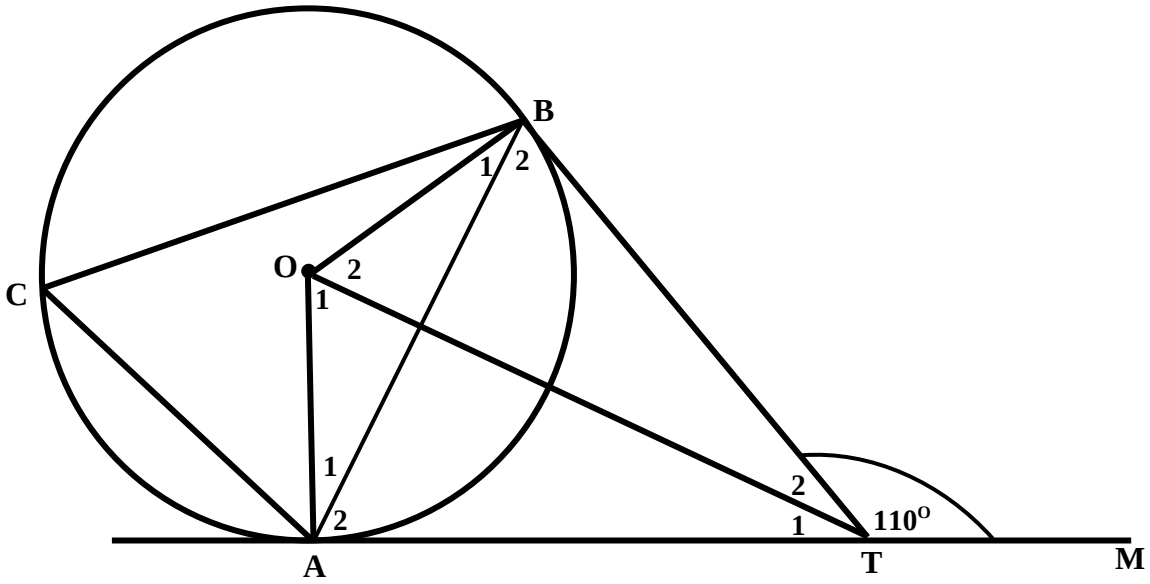
(a) $\hat{G}_1 = 90°$

(b) AH = 12

(c) OB = 13 (3)

9.1.2 Calculate the length of GH. (5)

9.2 In the diagram, O is the centre of the circle ACB. TA and TB are tangents to the circle at A and B respectively. O and T are joined. $B\hat{T}M = 110°$.



9.2.1 Prove that AOBT is a cyclic quadrilateral. (3)

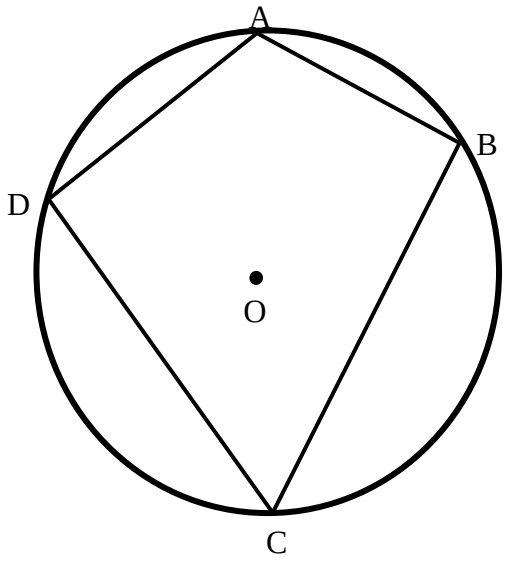
9.2.2 Give a reason why $\hat{T}_1 = \hat{T}_2$ (2)

9.2.3 Calculate $\hat{C}$. (4)

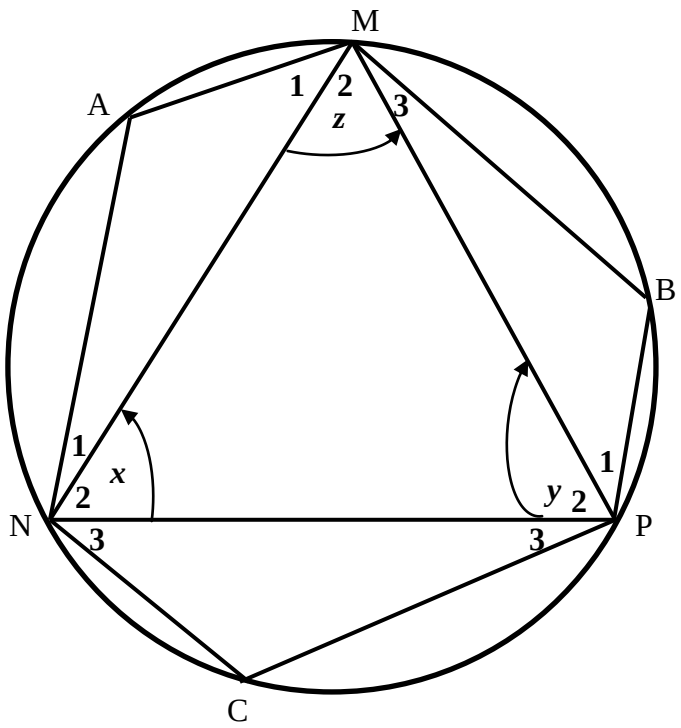
[17]

QUESTION 10

10.1 In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle. Use Euclidean Geometry methods to prove the theorem which states that $\hat{A} + \hat{C} = 180°$. (5)



10.2 In the diagram, the circle passes through A, M, B, P, C and N.



Let $\hat{N}_2 = x$, $\hat{P}_2 = y$ and $\hat{M}_2 = z$.

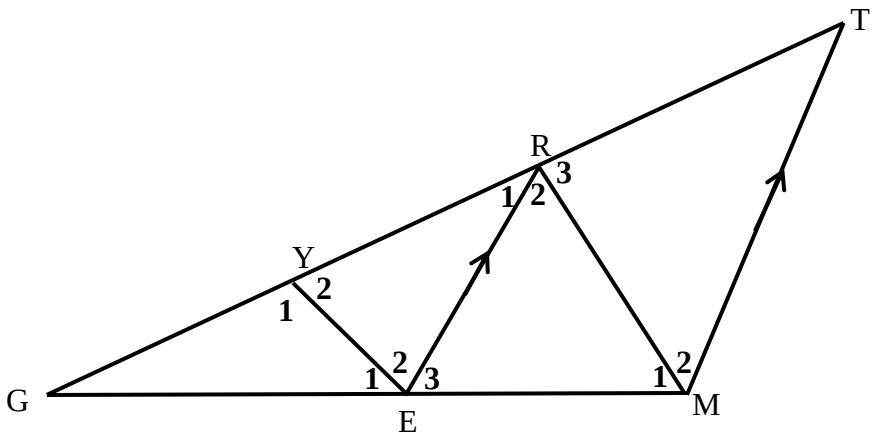
10.2.1 Write down the sizes of $\hat{A}$, $\hat{B}$ and $\hat{C}$ in terms of *x*, *y* and *z*. (2)

10.2.2 Hence, calculate the value of $\hat{A} + \hat{B} + \hat{C}$. (4)

[11]

QUESTION 11

In $\triangle$RGM, $\hat{E}_1 = \hat{R}_1 = \hat{R}_2$. T lies on RG produced so that TM || RE.



11.1 Give with reasons, TWO other angles which are equal to $\hat{R}_1$. (4)

11.2 Prove that $\frac{EM}{EG} = \frac{RM}{RG}$. (4)

11.3 Prove that $\Delta GYE \,|||\, \Delta GER$. (4)

11.4 Hence, prove that $\frac{EG}{EY} = \frac{RG}{RE}$. (1)

11.5 If it is further given that $G\hat{R}M = 90°$, RM = 6 and GM = 10.
Calculate the lengths of:

11.5.1 RG (2)

11.5.2 GE (4)
[19]

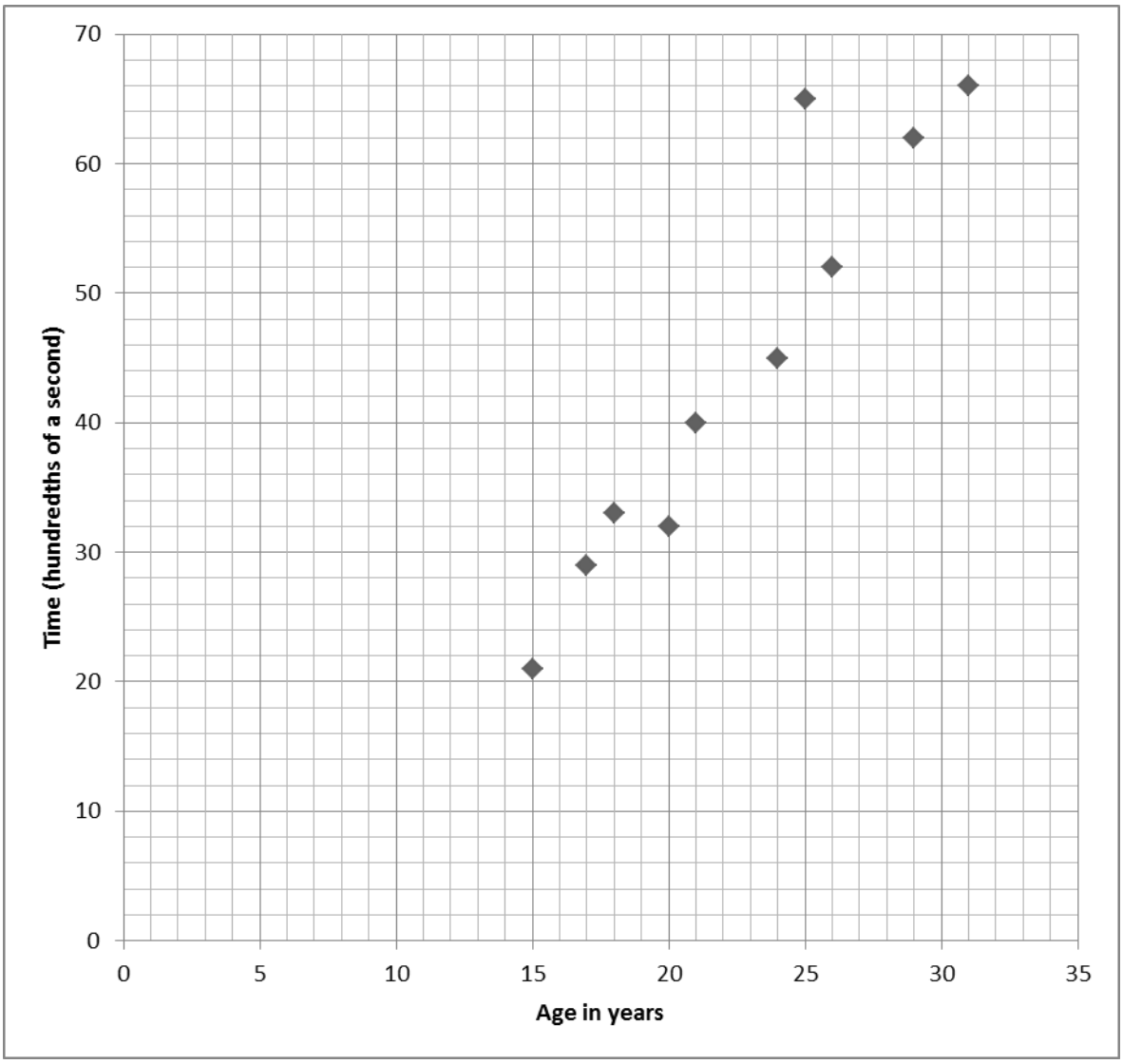
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NAME:

DIAGRAM SHEET 1

QUESTION 1.1



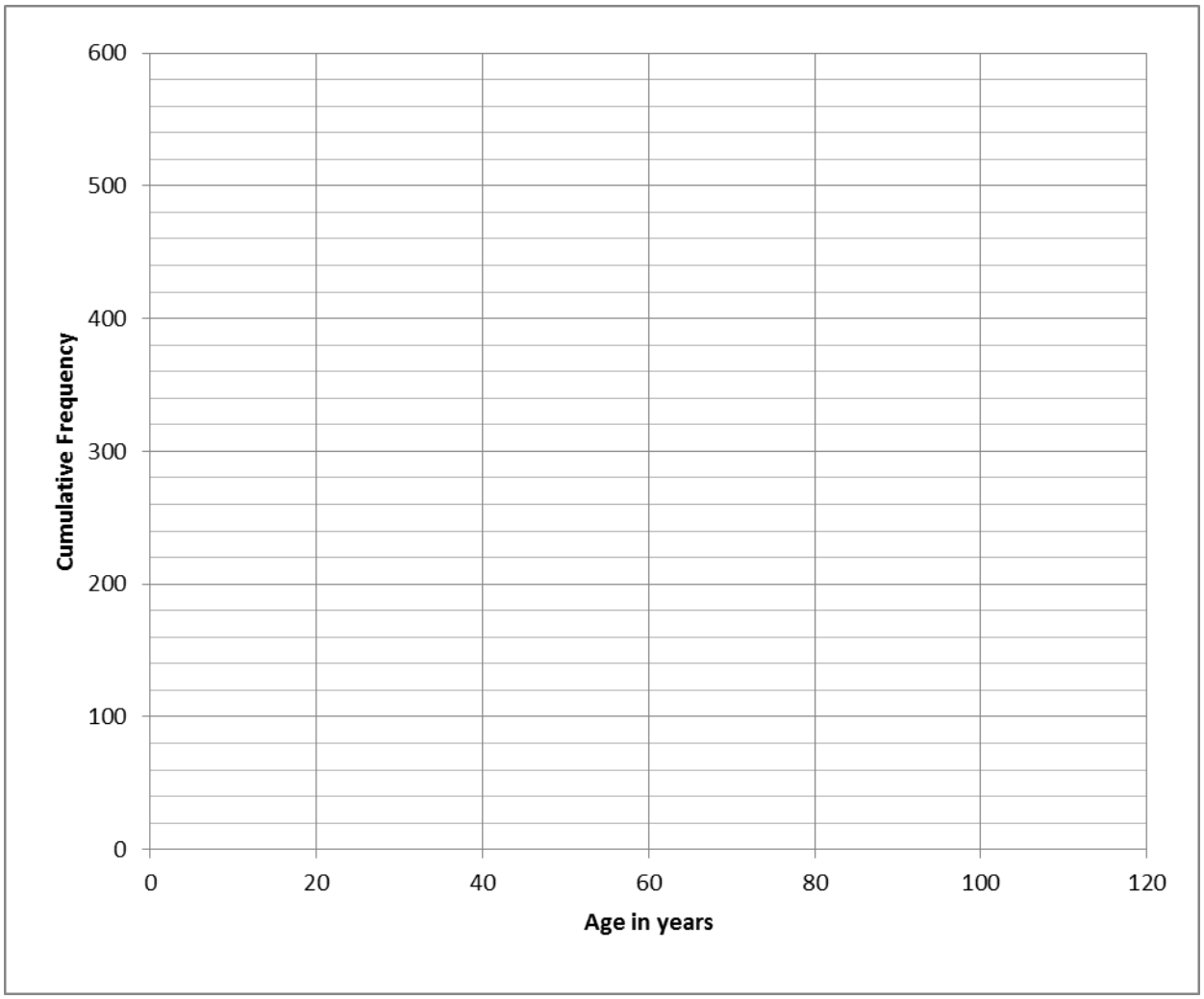
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DIAGRAM SHEET 2

QUESTION 2.1

Age in years	Number of people	Cumulative frequency
$0 \leq A < 10$	20	
$10 \leq A < 20$	130	
$20 \leq A < 30$	152	
$30 \leq A < 40$	92	
$40 \leq A < 60$	86	
$60 \leq A < 80$	18	
$80 \leq A < 100$	2	

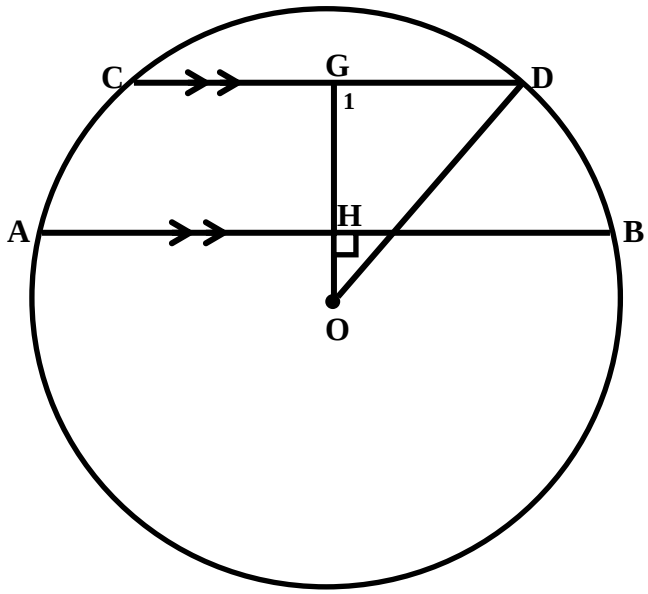
QUESTION 2.2 AND 2.3



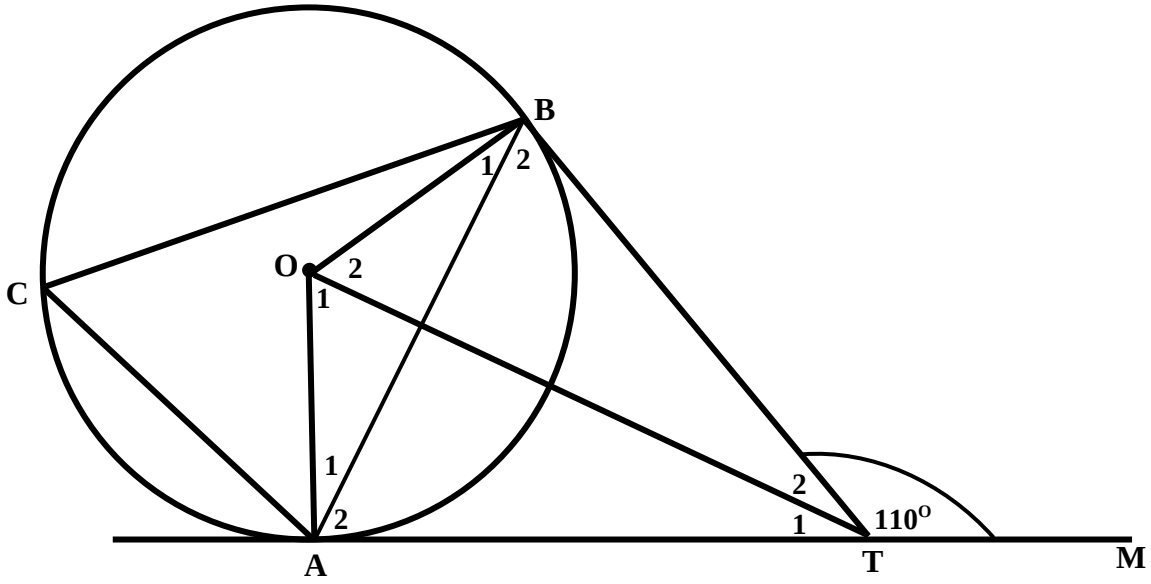
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DIAGRAM SHEET 3

QUESTION 9.1



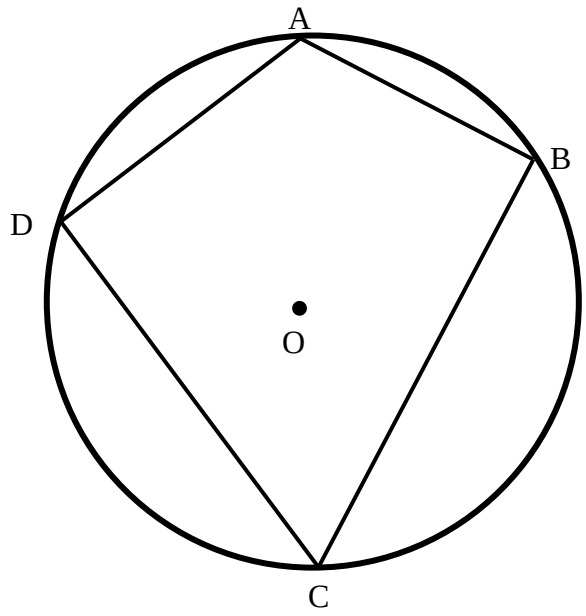
QUESTION 9.2



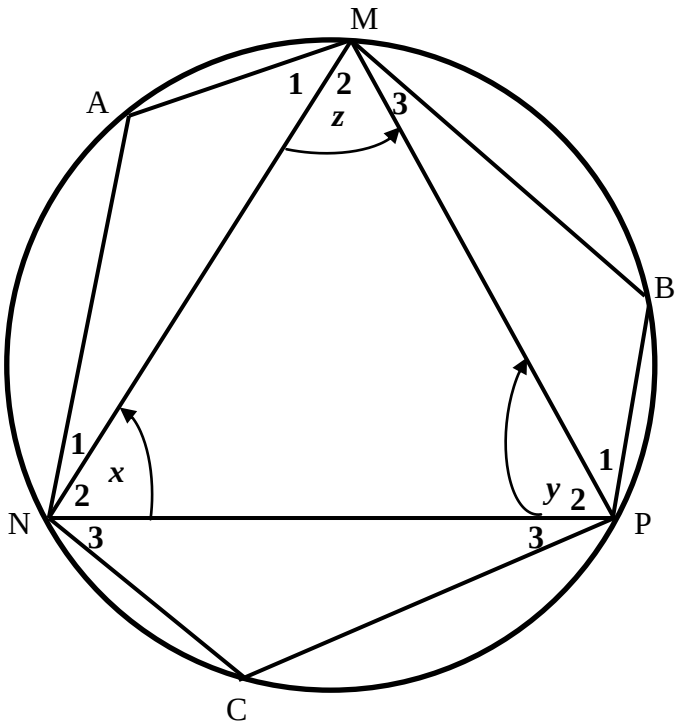
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DIAGRAM SHEET 4

QUESTION 10.1



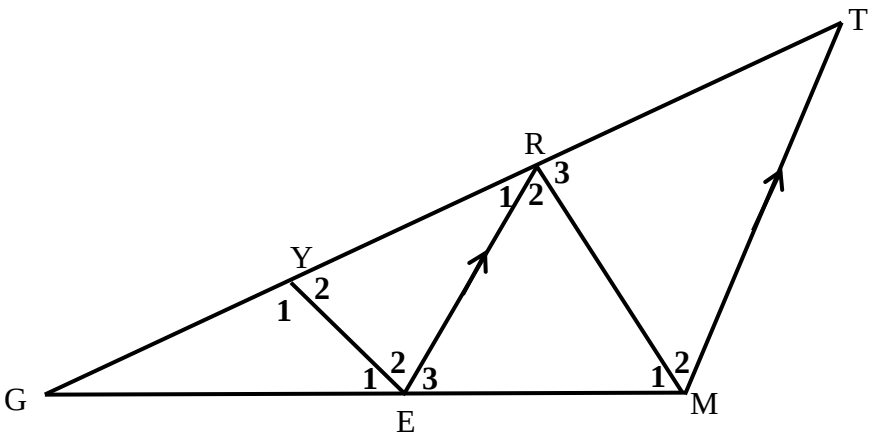
QUESTION 10.2



NAME:

DIAGRAM SHEET 5

QUESTION 11



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x\,;\,y) \to (x\cos\theta - y\sin\theta\,;\, y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$