GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION

2022

11091

TECHNICAL MATHEMATICS

PAPER 1

TIME: **3 hours**

MARKS: **150**

9 pages + 2 answer sheets and 2 information sheets

11091E

X05

INSTRUCTIONS AND INFORMATION

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $3\,x(x-4)=0$ (2)

1.1.2 $-2x-10=3x^2$ (Correct to one decimal place) (3)

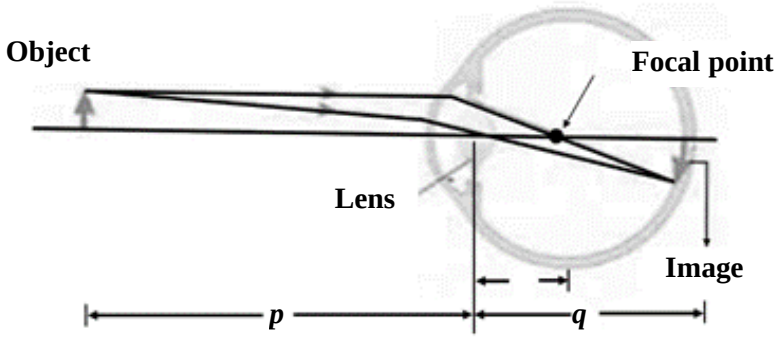
1.1.3 $-2x^2+3x\geq -2$ (3)

1.2 Solve for x and y simultaneously if:

$2x+y=3$ and $2y+2x^2-5=-x$ (6)

1.3 An equation relating the focal length (f) of a lens with the object distance (p) and the image distance (q) is given by the following equation:

$$f=\frac{pq}{p+q}$$



1.3.1 Make q the subject of the formula. (2)

1.3.2 Hence, calculate the numerical value of q if $p=5\ m$ and $f=0{,}2\ m$. (2)

1.4 Given the numbers A $=21$ and B $=5$.

1.4.1 Determine the product: A $\times$ B. (1)

1.4.2 Hence, convert your answer to QUESTION 1.4.1 to a binary number. (2)

[21]

QUESTION 2

2.1 The solutions of a quadratic equation are given by:

$$x = \frac{3 \pm \sqrt{t-3}}{4}$$

Determine the value(s) of t for which the equation has:

2.1.1 Real, rational roots (1)

2.1.2 Non-real roots (1)

2.2 Determine the value(s) of k for which the equation $x^2 - 4x + (k-1) = 0$ will have equal roots. (4)
[6]

QUESTION 3

3.1 Simplify the following WITHOUT the use of a calculator. (Show ALL calculations.)

3.1.1 $\left(\frac{\sqrt{b^3}}{b^{-\frac{1}{2}}}\right)^{-1}$ (3)

3.1.2 $\log_2 0{,}125 - 2\log_5 \sqrt{5} + \log_4 1$ (4)

3.1.3 $\left(\sqrt{3} - \sqrt{2}\right)^2$ (2)

3.2 Solve for x:

3.2.1 $\log_x 125 + \log_4 64 - \log_3 \frac{1}{3} = \log_2 128$ (4)

3.2.2 $27^{1-2x} - 243 = 0$ (3)

3.3 The current in amps, A, of an alternating current (AC) circuit is given by: $4{,}5\angle 30°$A. WITHOUT using a calculator, express the given current in rectangular form. (5)

3.4 Solve for x and y if:

$$x + iy = \frac{7+i}{2-i}$$ (5)
[26]

QUESTION 4

4.1 Consider the functions defined by:

$f(x) = -\frac{1}{2}x^2 + x + \frac{3}{2}$ and $g(x) = 3^x - 1$

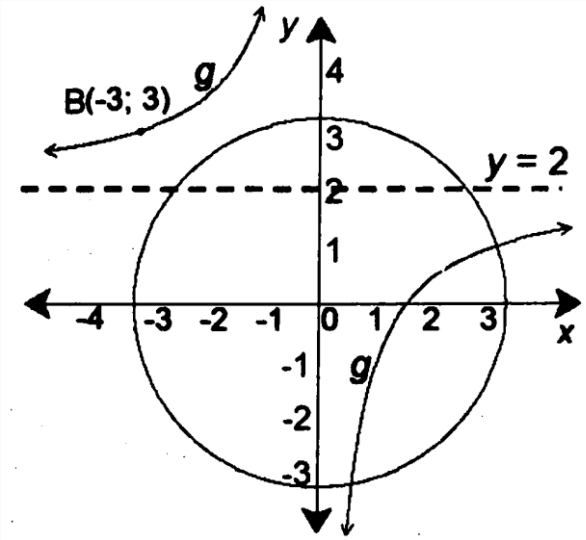
4.1.1 Determine:

(a) The y-intercept of f (1)
(b) The x-intercept of f (4)
(c) The coordinates of the turning point of f (4)
(d) The equation of the asymptote of g (1)
(e) The x-intercept of g (2)

4.1.2 Hence, sketch the graphs of f and g on the same set of axes on the ANSWER SHEET provided. Clearly show ALL the intercepts with the axes, turning points and any asymptote(s). (6)

4.1.3 For which value(s) of x is $g(x) < f(x)$ for $x > 0$? (1)

4.2 The graph of the circle $x^2 + y^2 = 10$ and $g(x) = \frac{a}{x} + k$ are shown in the figure below. $B(-3; 3)$ is a point on g.



4.2.1 Determine the values of a and k. (3)

4.2.2 Write down the radius of the circle in surd form. (1)

4.2.3 Write down the domain of the circle. (2)

4.2.4 Write down the range of g. (2)

[27]

QUESTION 5

5.1 The annual effective interest rate charged by a financial institution is 6,7% p.a. Calculate the nominal interest rate charged per annum if it is compounded monthly. (3)

5.2 A school bought a new welding machine for R6 500. The machine's value depreciates at a rate of 8% per annum to half its original value over a certain period.



5.2.1 Give the depreciated value of the welding machine at the end of the period. (1)

5.2.2 Determine how long it will take for the welding machine to depreciate to half its original value, using reduced balance depreciation. Give your answer to the nearest year. (3)

5.3 Mr. Nkosi invested an amount of R150 000 so that he could later buy safety clothing for his workers at his engineering company. Interest, compounded quarterly, is calculated at a rate of 10,5% p.a. for 5 years. At the end of the 33rd month, Mr. Nkosi withdrew an amount of R30 000 from the same account and then continued investing the balance for the remaining period. Determine the value of the investment at the end of the investment period. (6)

[13]

QUESTION 6

6.1 Determine $f'(x)$ using FIRST PRINCIPLES if $f(x) = 2x - 1$. (4)

6.2 Determine the following:

6.2.1 $f'(x)$ if $f(x) = \sqrt[3]{x^2} + 2x^5 - \frac{\pi}{x}$ (4)

6.2.2 $\frac{d}{dx}\left[\frac{(x-2)^2}{x^2}\right]$ (5)

6.3 Determine the average gradient of $f(x) = x^2 - 2x$ between the points where $x = 1$ and $x = -2$. (3)

6.4 Determine the equation of a tangent to the curve defined by $g(x) = 4x - x^2$ at the point where $x = 3$. (5)

[21]

QUESTION 7

Given: $f(x) = x^3 + 4x^2 - 3x - 18$

7.1 Show that $(x - 2)$ is a factor of $f(x)$. (2)

7.2 Hence, or otherwise, find the *x*-intercepts of f. (4)

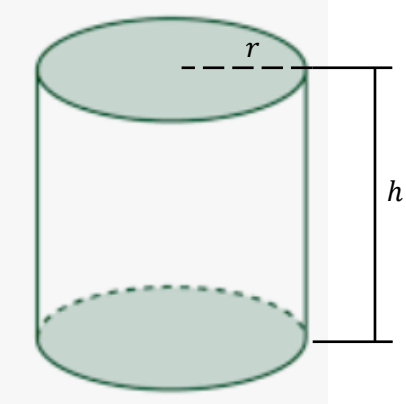
7.3 Determine the co-ordinates of the turning points of f. (5)

7.4 Sketch the graph of f on the ANSWER SHEET provided. Clearly show ALL the intercepts with the axes and the turning points. (4)

[15]

QUESTION 8

A cylindrical right prism has a radius of r cm and a height of h cm. The dimensions are restricted in such a way that the sum of the circumference of its base and its height is always equal to 21 cm.



The following formulae may be used:

Volume of a Cylinder $= \pi r^2 h$
Total Surface Area of a Cylinder $= 2\pi r^2 + 2\pi rh$

8.1 Show that the height of the cylinder can be expressed as:

$h = 21 - 2\pi r$ (1)

8.2 Hence, show that the volume of the cylinder is obtained by:

$V = 21\pi r^2 - 2\pi^2 r^3$ (2)

8.3 Determine r in terms of π if the volume is a maximum. (3)

8.4 Hence, determine the maximum volume of the cylinder in terms of π. (2)
[8]

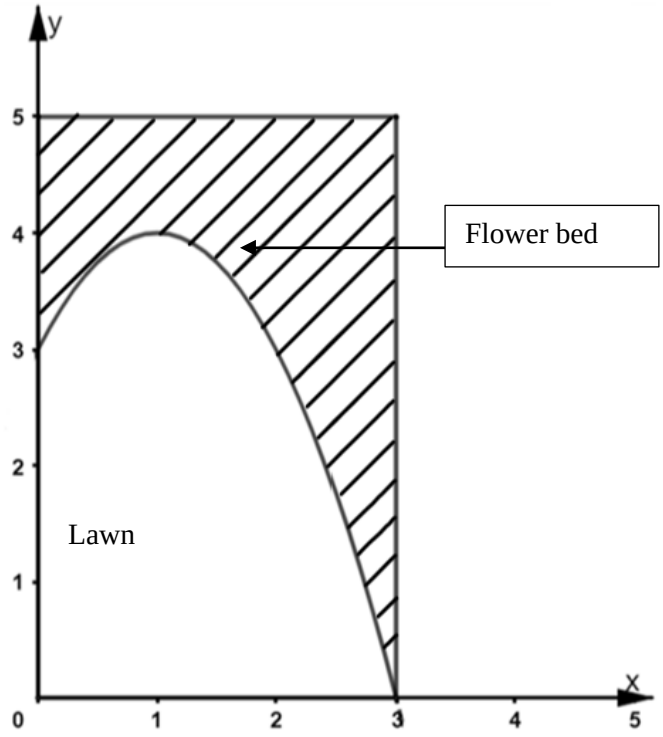
QUESTION 9

9.1 Determine the following integrals:

9.1.1 $\int \left(\frac{2}{5x} - 2^{3x} + 4\right) dx$ (3)

9.1.2 $\int \left(\frac{2a^3 - 54}{a - 3}\right) da$ (5)

9.2 A gardener is considering a new design for his client's garden. He has a rectangular piece of lawn measuring 5 m by 3 m, and he wants to dig up a part of it to include a flower bed. He draws a plan of the lawn and flower bed on a piece of graph paper, taking the bottom and left-hand edges as the axes, and chooses a scale such that 1 unit along each axis represents 1 metre on the ground.



The equation of the curved edge of the flower bed is given by:

$y = -x^2 + 2x + 3$

Calculate the area of the flower bed. (5)

[13]

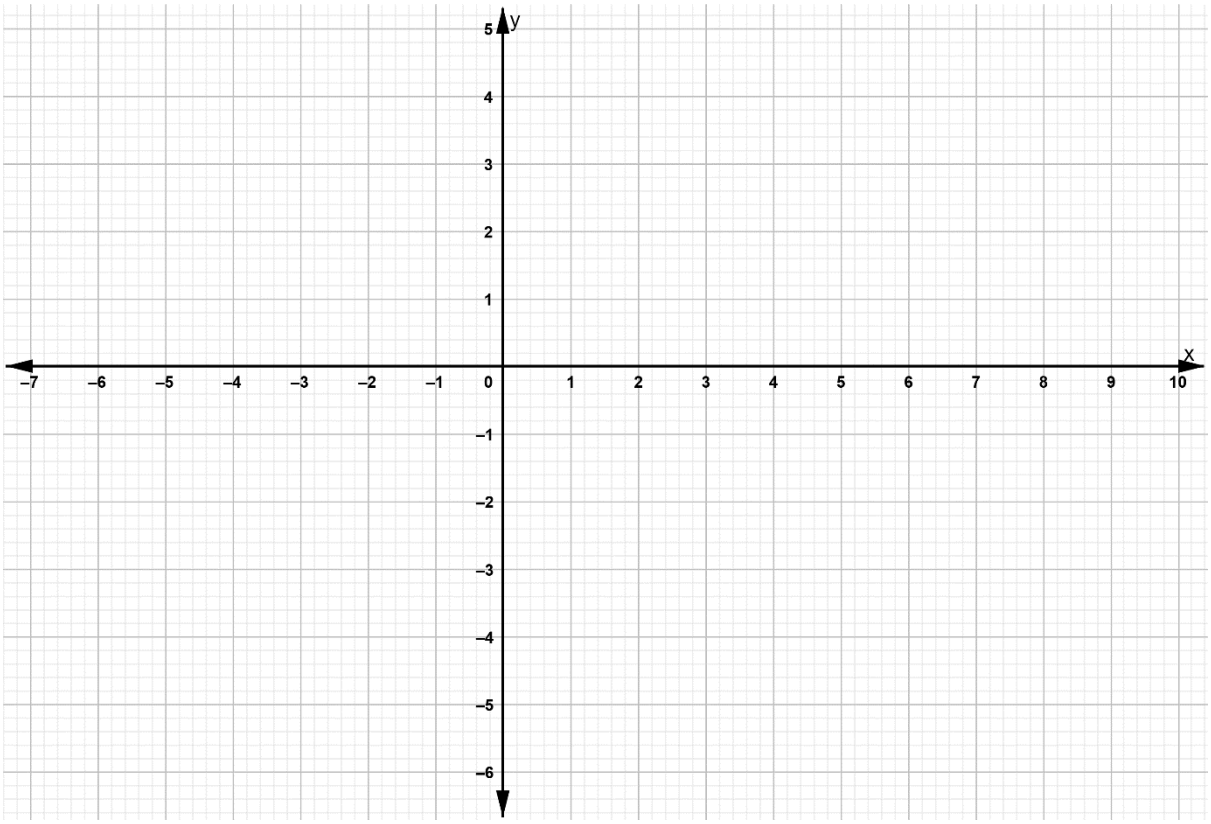
TOTAL: 150

END

ANSWER SHEET

Candidate's Name: _____

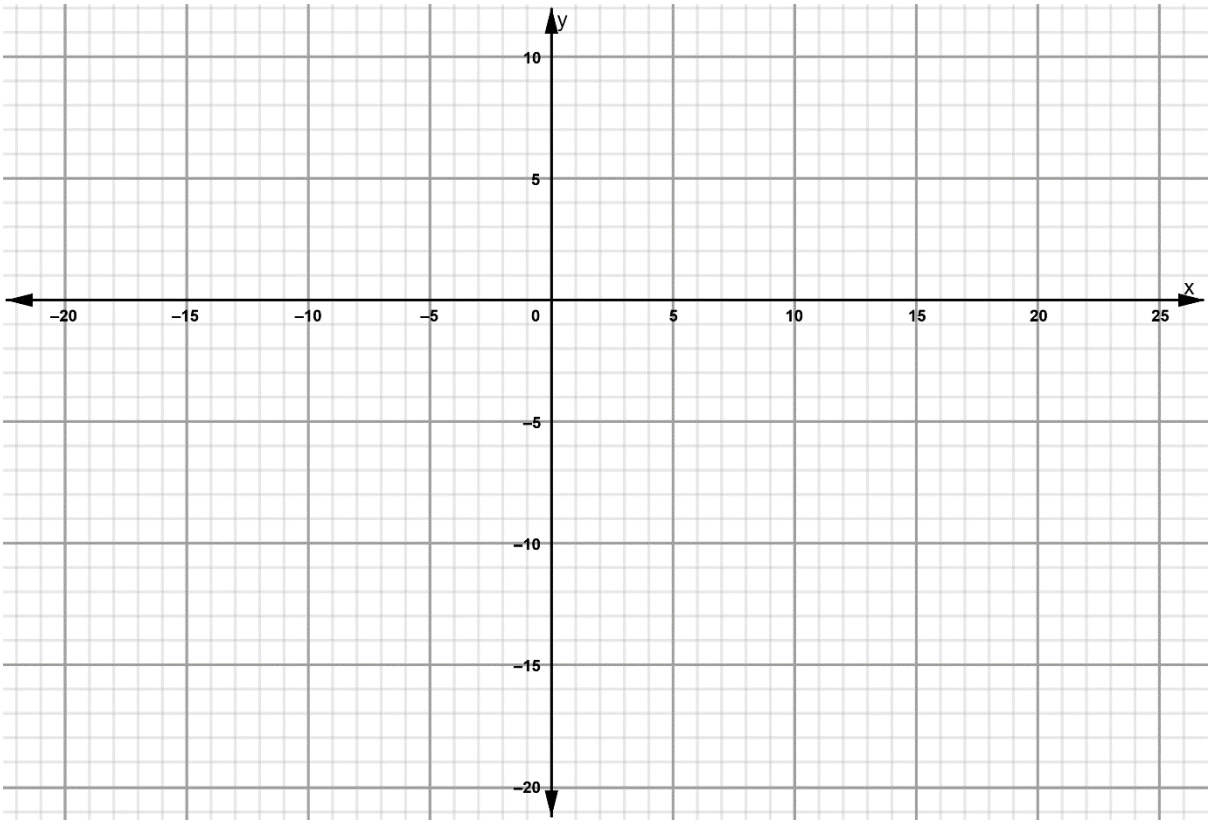
QUESTION 4.1.2



ANSWER SHEET

Candidate's Name: _____

QUESTION 7.4



INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = log_a\, b, \quad a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$i_{eff} = \left(1 + \frac{i^m}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1 \qquad \int kx^n dx = k.\frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int \frac{1}{x} dx = ln(x) + C, x > 0 \qquad \int \frac{k}{x} dx = k.ln(x) + C, x > 0$$

$$\int a^x dx = \frac{a^x}{ln\, a} + C, a > 0 \qquad \int ka^{nx} dx = k.\frac{a^{nx}}{n\, ln\, a} + C, a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$

$$\text{Area of } \Delta\text{ABC} = \frac{1}{2}ab \cdot \sin \text{C}$$

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$ $\quad \cot^2\theta + 1 = \text{cosec}^2\theta$ $\quad \pi rad = 180°$

Angular velocity $= \omega = 2\pi n$ where n = rotation frequency
Angular velocity $= \omega = 360^0 n$ where n = rotation frequency

Circumferential velocity $= v = \pi Dn$ where D = diameter and n = rotation frequency

Arc length $s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \frac{rs}{2}$ where r = radius, s = arc length
Area of a sector $= \frac{r^2\theta}{2}$ where r = radius, θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + \ldots + m_n)$ where a = equal parts, $m_1 = \frac{o_1 + o_2}{2}$ and n = number of ordinates

OR

$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where a = equal parts, $o_n = n^{th}$ ordinate and n = number of ordinates