GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION/*JUNIE EKSAMEN*
GRADE/*GRAAD* 12

2023

MARKING GUIDELINES/*NASIENRIGLYNE*

TECHNICAL MATHEMATICS/*TEGNIESE WISKUNDE*

(PAPER/*VRAESTEL* 1)

Marking Codes/*Nasienkodes*	
A	Accuracy/*Akkuraatheid*
CA	Consistent accuracy/*Volgehoue Akkuraatheid*
M	Method/*Metode*
R	Rounding/*Afronding*
NPR	No penalty for rounding/*Geen straf vir afronding*
NPU	No penalty for units omitted/*Geen straf vir weggelate eenhede*
S	Simplification/*Vereenvoudiging*
SF	Substitution in correct formula/*Vervanging in korrekte formule*
ST	Statement/*Stelling*
ST/RE	Statement and Reason/*Stelling en Rede*
RE	Reason/*Rede*

NOTE/LET WEL:

- If a candidate answers a question TWICE, mark only the FIRST attempt/
  Indien 'n kandidaat 'n vraag TWEE keer beantwoord, sien slegs die eerste poging na.
- If a candidate has crossed out an attempt to answer the question and did not redo it, mark the crossed-out version/
  Indien 'n kandidaat 'n antwoord doodgetrek het, maar dit nie oordoen het nie, merk die doodgetrekte weergawe.
- Consistent accuracy applies in all aspects of the marking guidelines, where indicated.
  Volgehoue akkuraatheid word toegepas deur die hele nasienriglyne en waar aangedui.

QUESTION/*VRAAG* 1					CL
1.1	1.1.1	$x^2 + 25 = 0$ $x^2 = -25$ $x = \sqrt{-25}$ $x = 5i$ **or/*of*** $x = -5i$ **AO 3/3**	✓ $x^2 = -25$ A ✓ $5i$ CA ✓ $-5i$ CA	(3)	1D
	1.1.2	$-3x^2 = -2x - 2$ $0 = 3x^2 - 2x - 2$ ✓A $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-2)}}{2(3)}$ $x = \frac{2 \pm \sqrt{28}}{6}$ $x = 1,22$ **or/*of*** $x = -0,55$ **OR/*OF*** $-3x^2 + 2x + 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-2)}}{2(3)}$ $x = \frac{2 \pm \sqrt{28}}{6}$ $x = 1,22$ **or/*of*** $x = -0,55$	✓ S A ✓ SF CA ✓ $x = 1,22$ CA ✓ $x = -0,55$ CA ✓ S A ✓ SF CA ✓ $x = 1,22$ CA ✓ $x = -0,55$ CA	(4)	2M

	1.1.3	$x^2 - 2x - 15 \leq 0$ $(x + 3)(x - 5) \leq 0$ $x = -3$ or/*of* $x = 5$ $-3 \leq x \leq 5$ **OR/*OF*** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-15)}}{2(1)}$ $x = \frac{2 \pm \sqrt{64}}{2}$ $x = -3$ **or/*of*** $x = 5$ $-3 \leq x \leq 5$	✓ Factors/*Faktore* CA ✓ Notation/*Notasie* A ✓ Endpoints/*Eindpunte* CA ✓ SF A ✓ Notation/*Notasie* A ✓ Endpoints/*Eindpunte* CA	(3)	2M
1.2	$2x - 1 = y$ and/*en* $x^2 - xy = 3x - 3$ $x^2 - x(2x - 1) = 3x - 3$ $x^2 - 2x^2 + x - 3x + 3 = 0$ $-x^2 - 2x + 3 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-1)(3)}}{2(-1)}$ $x = \frac{2 \pm \sqrt{16}}{-2}$ $x = -3$ **or/*of*** $x = 1$ $2(-3) - 1 = y$ **or/*of*** $2(1) - 1 = y$ $y = -7$ **or/*of*** $y = 1$	✓ SF A ✓ Standard form *Standaard vorm* CA ✓ SF A ✓ Both x-values *Beide x-waardes* CA ✓ Both y-values *Beide y-waardes* CA	(5)	

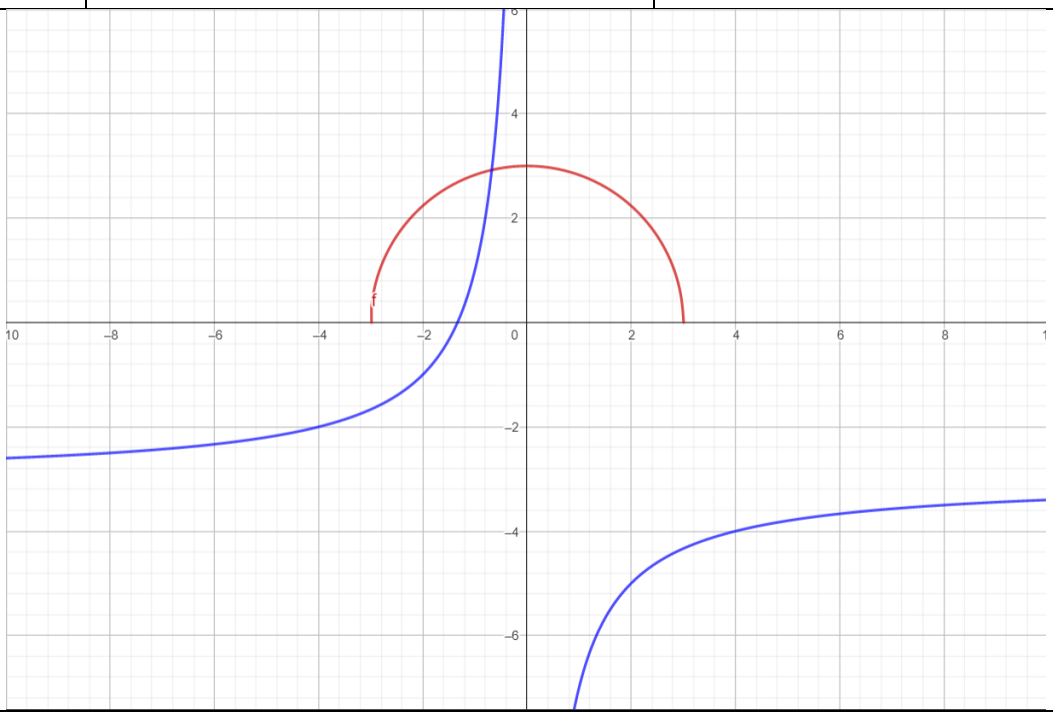
1.3	1.3.1	$N = N_0 e^{kt}$ $\frac{N}{e^{kt}} = N_0$	✓ divide e^{kt}/*deel met* e^{kt} M ✓ Answer/*Antwoord* CA	(2)	1D
	1.3.2	$100\ 000 = 6\ 000\ e^{0,065t}$	✓ Substitution/*Substitusie (100 000)* CA ✓ Substitution/*Substitusie (6 000) en (0,065)* CA	(2)	1E
1.4	1.4.1	$100111_2 + 10000_2 = 110111_2$	✓ M ✓ Answer/*Antwoord* CA	(2)	1E
	1.4.2	$110111_2 = 55$	✓ Answer/*Antwoord* CA	(1)	1E
				[22]	

QUESTION/*VRAAG* 2					
2.1	2.1.1	$10 - 20k = 0$ $-20k = -10$ $k = \frac{1}{2}$	✓ $10 - 20k = 0$ ✓ Answer/*Antwoord* CA	(2)	1E
	2.1.2	$10 - 20k < 0$ $-20k < -10$ $k > \frac{1}{2}$	✓ $10 - 20k < 0$ ✓ Answer/*Antwoord* CA	(2)	1D
2.2	$x^2 - 5x - 7 = 0$ $\Delta = b^2 - 4ac$ $\Delta = (-5)^2 - 4(1)(-7)$ $= 53$ $\Delta > 0$ Not perfect square, ∴ the roots are real, unequal and irrational. *Nie 'n volkome vierkant nie ∴ wortels is reël, ongelyk en irrasionaal.*		✓ Formula/*Formule* ✓ Substitution/*Substitusie* A ✓ Answer/*Antwoord* CA ✓ Reason/*Rede* A	(4)	2D
				[8]	

QUESTION/*VRAAG* 3					
3.1	3.1.1	$\left(729\ x^{-6}\right)^{-\frac{1}{6}}$ $= \left(3^6 x^{-6}\right)^{\frac{-1}{6}}$ $= 3^{-1}.x^1$ $= \frac{1}{3}x$	✓ $3^{-1}.x^1$ ✓ Answer/*Antwoord* CA	(2)	2D
	3.1.2	$\frac{\sqrt{5}}{1-\sqrt{5}}$ $= \frac{\sqrt{5}}{1-\sqrt{5}} \times \frac{1+\sqrt{5}}{1+\sqrt{5}}$ $= \frac{\sqrt{5}\left(1+\sqrt{5}\right)}{1-5}$ $\frac{\sqrt{5}+5}{-4}$	✓ Multiply/*Vermenigvuldig* $\frac{1+\sqrt{5}}{1+\sqrt{5}}$ M ✓ Simplification/ *Vereenvoudiging* CA ✓ Answer/*Antwoord* CA **AO** $\frac{1}{3}$	(3)	2D
3.2	3.2.1	$\log 1 \times \log 10 \times \log 100$ $= 0 \times \log 10 \times \log 100$ $= 0$	✓ $\log 1 = 0$ ✓ Answer/*Antwoord* A **AO** $\frac{0}{2}$	(2)	1D
	3.2.2	$\log_3\left(27 \times 81\right)$ $= \log_3\left(3^3 . 3^4\right)$ $= \log_3 3^7$ $= 7\log_3 3^3$ $= 7(1)$ $= 7$	✓ $\log_3 3^7$ A ✓ log rule/*wet* M ✓ Answer/*Antwoord* A **AO** $\frac{1}{3}$	(3)	2M

3.3	$\log(x+2)+\log(x-1)=1$ $\log(x+2)(x-1)=\log 10$ $x^2+2x-1x-2=10$ $x^2+x-12=0$ $(x-3(x+4)=0$ $x=3$ **or/*of*** $x\neq -4$		✓ Log rule/*wet* M ✓ S ✓ $x^2+x-12=0$ CA ✓ $x=3$ CA ✓ $x\neq -4$ CA	(5)	2D
3.4	3.4.1	z = 2*w* – 7*i* $w=\frac{1}{2}+3i$ $z=2\left(\frac{1}{2}+3i\right)-7i$ z = 1 + 6*i* – 7*i* z = 1 – *i*	✓ Substitution/*Substitusie* A ✓ Answer/*Antwoord* CA	(2)	2E
	3.4.2	r (mod) $=\sqrt{a^2+b^2}$ r (mod) $=\sqrt{(1)^2+(-1)^2}$ r (mod) $=\sqrt{2}$ r (mod) $=1{,}4$	✓ Answer/*Antwoord* CA	(1)	2E
	3.4.3	$\theta=360°-\tan^{-1}\frac{b}{a}$ $\theta=360°-\tan^{-1}\left(\frac{1}{1}\right)$ $\theta=315°$ **OR/*OF*** $\tan\theta=\frac{b}{a}$ $\theta=\tan^{-1}\left(-\frac{1}{1}\right)$ $\theta=45°$ $\theta=360°-45°$ $=315°$	✓ Answer/*Antwoord* CA	(1)	2D
	3.4.4	z = 1,4 cis 315°/$\sqrt{2}$ cis 315°	✓ Answer/*Antwoord* CA	(1)	1M
3.5	$x+i=(1-yi)(2+3i)$ $x+i=2+3i-2yi-3yi^2$ $x+i=2+3y+3i-2yi$ $x=2+3y$ And/*En* $1=3-2y$ $y=1$ And/*En* $x=2+3(1)=5$		✓ Multiply/*Vermenigvuldig* M ✓ Simplification/ *Vereenvoudiging* CA ✓ $y=1$ ✓ $x=5$	(4)	3D
				[24]	

QUESTION/*VRAAG* 4					
4.1	4.1.1	$f(x) = x^2 - 6x - 16$ $0 = x^2 - 6x - 16$ $0 = (x - 8)(x + 2)$ $A(-2; 0)$ and/*en* $B(8; 0)$	✓ $0 = x^2 - 6x - 16$ A ✓ Factors/*Faktore* CA ✓ $A(-2; 0)$ ✓ $B(8; 0)$ **AO** $\frac{2}{4}$	(4)	1D
	4.1.2	$x = -\frac{b}{2a}$ $x = -\frac{-6}{2(1)}$ $x = 3$ $f(3) = (3)^2 - 6(3) - 16$ $= -25$ $E(3; -25)$	✓ $x = -\frac{b}{2a}$ ✓ Substitution/*Substitusie* A ✓ $x = 3$ CA ✓ y = –25 CA **AO** $\frac{4}{4}$	(4)	2D
	4.1.3	$f(x) = g(x)$ $x^2 - 6x - 16 = x - 8$ $x^2 - 6x - 16 - x + 8 = 0$ $x^2 - 7x - 8 = 0$ $0 = (x - 8)(x + 1)$ $x \neq 8$ $x = -1$ y = –9 $P(-1; -9)$	✓ $x^2 - 6x - 16 = x - 8$ A ✓ $x^2 - 7x - 8 = 0$ CA ✓ Factors/Faktore CA ✓ Both x-values/ *Beide x-waardes* CA ✓ y-value/*y-waardes* CA ✓ $P(-1; -9)$	(6)	3D
	4.1.4	AB = 10 units/*eenhede*	✓ Answer/*Antwoord* CA	(1)	1E
	4.1.5	$x \in (-1; 8)$	✓ Notation/*Notasie* CA ✓ Endpoints/*Eindpunte* CA	(2)	2D
4.2	4.2.1	$y = -4$	✓ Answer/*Antwoord* CA	(1)	1E
	4.2.2	$0 = 2^x - 4$ $2^x = 4$ $2^x = 2^2$ $x = 2$ $P(2; 0)$	✓ $f(x) = 0$ M ✓ $P(2; 0)$ CA	(2)	1D

	4.2.3	$y = 2^0 - 4$ $y = 1 - 4$ $y = -3$ $R(0; -3)$	✓ $x = 0$ M ✓ $R(0; -3)$ CA	(2)	2E
	4.2.4	$y > -4$	✓ Notation/*Notasie* A ✓ Endpoint/*Eindpunt* CA	(2)	2D
4.3					
			For/*Vir g(x)* ✓ Shape/*Vorm* ✓ Asymptotes/*Asimptote* ✓ Intercept/*Afsnit* For/*Vir f(x)* ✓ Shape/*Vorm* ✓ y-intercept/*y-afsnit* ✓ x-intercept/*x-afsnit*	(6)	2E
				[30]	

QUESTION/*VRAAG* 5					
5.1	5.1.1	B The growth amount is constant, hence a straight line graph will represent the growth of the investment. *Die groei waarde is konstant, dus stel 'n reguitlyn grafiek die groei van die belegging voor.*	✓ Answer/*Antwoord* CA ✓ Reason/*Rede* A	(2)	1D
	5.1.2	A The growth amount is increasing every year because interest is earned on interest as well. Therefore the growth of the investment is represented by a curved graph. *Die groei waarde verhoog elke jaar want rente word op rente verdien. Dus word die groei deur n kurwe grafiek voorgestel.*	✓ Answer/*Antwoord* CA ✓ Reason/*Rede* A	(2)	1D
5.2	5.2.1	A = P(1 + in) $18\ 000 = 12\ 000\left(1+\frac{7,5}{100}n\right)$ $\frac{18\ 000}{12\ 000}=\left(1+\frac{7,5}{100}n\right)$ $\frac{18\ 000}{12\ 000}=\left(1+\frac{7,5}{100}n\right)$ $\frac{\frac{18\ 000}{12\ 000}-1}{0,075}=n$ $n = 6\frac{2}{3}$ n = 7 years/*jaar*	✓ Formula/*Formule* A ✓ SF A ✓ Answer/*Antwoord* CA	(3)	2D

	5.2.2	Jan 2018 – Jan 2020: $A = P(1 + i)^n$ $= 12\ \ 000\left(1+\frac{0,0696}{12}\right)^{2\times 12}$ = 13 786,70 Jan 2020 – Dec/*Des* 2021: P = 13 786,70 – 3 800 = 9 986,70 $A = 9\ 986,70\left(1+\frac{0,0696}{12}\right)^{2\times 12}$ A = R11 473,64	✓ Formula/*Formule* ✓ SF A ✓ Answer/*Antwoord* CA ✓ M CA ✓ SF A ✓ Answer/*Antwoord* CA	(6)	2D
5.3	$i_{eff} = \left(1+\frac{i}{m}\right)^m - 1$ $i_{eff} = \left(1+\frac{0,098}{12}\right)^{12} - 1$ i_{eff} = 0,1025233891 = 10,25%	✓ $\left(1+\frac{0,098}{12}\right)^{12}$ SF/A ✓ 0,102523891 CA ✓ 10,25% CA **NPR**	(3)	2D
5.4	$A = P(1 - i)^n$ $11\ 090,41 = 120\ 000(1 - i)^{12}$ $\sqrt[12]{\frac{11\ \ 090\ ,41}{120\ \ 000}} = 1 - i$ $\sqrt[12]{\frac{11\ \ 090\ ,41}{120\ \ 000}} - 1 = -i$ –0,180001643 = –i i = 18%	✓ Formula/*Formule* A ✓ SF A ✓ Simplification/ *Vereenvoudiging* CA ✓ Answer/*Antwoord* CA	(4)	3M
			[20]	

QUESTION/*VRAAG* 6					
6.1	$f(x) = 2x + 3$ $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0}\frac{2(x+h)+3-(2x+3)}{h}$ $= \lim_{h\to 0}\frac{2x+2h+3-2x-3}{h}$ $= \lim_{h\to 0}\frac{2h}{h}$ $= \lim_{h\to 0}\frac{h(2)}{h}$ $= \lim_{h\to 0} 2$ $= 2$		✓ Formula/*Formule* A ✓ SF A ✓ Simplification/ *Vereenvoudiging* CA ✓ $= \lim_{h\to 0}\frac{2h}{h}$ CA ✓ Answer/*Antwoord* CA	(5)	2D
6.2	6.2.1	$y = -x^{-5} + 3x^4$ $\frac{dy}{dx} = 5x^{-6} + 12x^3$ $\frac{dy}{dx} = \frac{5}{x^6} + 12x^3$	✓ $5x^{-6}/\frac{5}{x^6}$ CA ✓ $12x^3$ CA	(2)	2M
	6.2.2	$f(x) = \frac{3}{x^4} - \frac{x}{\sqrt{x}}$ $f(x) = 3x^{-4} - x^{\frac{1}{2}}$ $f'(x) = -12x^{-5} - \frac{1}{2}x^{-\frac{1}{2}}$ $f'(x) = \frac{-12}{x^5} - \frac{1}{2\sqrt{x}}$	✓ $3x^{-4}$ S ✓ $-x^{\frac{1}{2}}$ S ✓ $-12x^{-5}/\frac{-12}{x^5}$ ✓ $-\frac{1}{2}x^{-\frac{1}{2}}/\frac{1}{2\sqrt{x}}$	(4)	3E
	6.2.3	$D_x\left[\frac{x^2+x-6}{x+3}\right]$ $= D_x\left[\frac{(x+3)(x-2)}{x+3}\right]$ $= D_x(x-2)$ $= 1$	✓ Factors/*Faktore* CA ✓ $(x-2)$ S ✓ Answer/*Antwoord* CA	(3)	3E

6.3	$f(0) = 2$ $f(2) = -6$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{2-(-6)}{0-2}$ $m = -4$ **OF/*OR*** Average gradient/*Gemiddelde gradiënt* $= \frac{f(b) - f(a)}{b-a}$ $= \frac{-2(0)^2 + 2 - \left((-2)2^2 + 2\right)}{0-2}$ $= \frac{2-(-6)}{-2}$ $= -4$	✓ $f(0) = 2$ & $f(2) = -6$ A ✓ SF CA ✓ Answer/*Antwoord* CA ✓ ✓ $f(0) = 2$ & $f(2) = -6$ A ✓ SF CA ✓ Answer/*Antwoord* CA	(3)	2D
			[17]	

QUESTION/*VRAAG* 7				
7.1	(0 ; –5)	✓ –5	(1)	1M
7.2	$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$ ✓CA $f(-1) = 0$ ✓CA ∴ $(x + 1)$ is a factor/*is 'n faktor*.	✓ SF A ✓ $f(-1) = 0$ A	(2)	1E
7.3	Synthetic division/*Sintetiese deling* $x = -1$ $(x + 1)(x^2 - 4x - 5) = 0$ $(x + 1)(x + 1)(x - 5) = 0$ x = –1 **or/*of*** $x = -1$ **or/*of*** $x = 5$ (–1; 0) (–1; 0) (5; 0) D (5: 0)	✓ Division/*Deling* ✓ Factors/Faktore CA ✓ Answer/*Antwoord* CA	(3)	3M
7.4	$f'(x) = 3x^2 - 6x - 9$ $0 = 3x^2 - 6x - 9$ $0 = x^2 - 2x - 3$ $(x - 3)(x + 1) = 0$ **OR/*OF*** $x = \frac{-6 \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-6 \pm \sqrt{(-6)^2 - 4(3)(-9)}}{2(3)}$ $x = 3$ **or/*of*** $x = -1$ $f(3) = (3)^3 - 3(3)^2 - 9(3) - 5$ y = –32 $f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$ $y = 0$ ∴ (3; –32) and/*en* (–1 ; 0) ∴ *C*(3; –32)	✓ $f'(x)$ A ✓ $f'(x) = 0$ A ✓ Both x-values *Beide x-waardes* CA ✓ Both y-values *Beide y-waardes* CA ✓ (3; –32)	(5)	4M
7.5	$-\infty < x < -1$ **OR/*OF*** $x > 3$ ✓CA	✓ Notation/*Notasie* CA ✓ Endpoints/*Eindpunte* CA	(2)	4D
			[13]	

QUESTION/*VRAAG* 8				
8.1	I = price per item × total items sold *I = prys per item × totaal items verkoop* $I = (0{,}15 - 0{,}000002x) \times x$ $I = 0{,}15 - 0{,}000002x^2$	✓ SF A ✓ Answer/*Antwoord* CA	(2)	2D
8.2	$I = 0{,}15x - 0{,}000002x^2$ $I = 0{,}15(5000) - 0{,}000002(5000)^2$ I = R700	✓ SF A ✓ Answer/*Antwoord* CA	(2)	2D
8.3	$I' = 0{,}15 - 0{,}000004x$ ✓CA $0 = 0{,}15 - 0{,}000004x$ ✓CA $-0{,}15 = -0{,}000004x$ x = R37 500 ✓CA	✓ I' A ✓ $I' = 0$ A ✓ Answer/*Antwoord* CA	(3)	4E
			[7]	

QUESTION/*VRAAG* 9				
9.1	$\int \left(x^{-2} + \frac{1}{x} \right) dx$ $= \frac{x^{-2+1}}{2 + 1} + 1n\ x + c$ $= -\frac{1}{x} + 1n\ x + c$	✓ $-\frac{1}{x}$ A ✓ In x A ✓ + c A	(3)	2D
9.2	$\int_{-1}^{1} (-3x^2 + 3)\ dx$ ✓CA $= \left[-x^3 \text{✓}^{CA} + 3x \text{✓}^{CA} \right]_{-1}^{1}$ ✓CA $= \left[-(1)^3 + 3(1) \right] - \left[-(-1)^3 + 3(-1) \right]$ $= [2] - [-2]$ $= 4$ units^2	✓ $\int_{-1}^{1}(-3x^2 + 3)\ dx$ A ✓ $-x^3$ A ✓ $3x$ A ✓ Substitute/*Vervang* 1 A ✓ Substitute/*Vervang* –1 A ✓ Answer/*Antwoord* CA	(6)	2D
			[9]	
			Total/*Totaal:* 150	