education

Department of
Education
FREE STATE PROVINCE

PREPARATORY EXAMINATION

GRADE 12

MATHEMATICS P1

SEPTEMBER 2018

TIME: 3 HOURS

MARKS: 150

This question paper consists of 9 pages and 1 information sheet.

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number your answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. An approved scientific calculator (non-programmable, non-graphic) may be used, unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

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QUESTION 1

1.1 Solve for x:

1.1.1 $-3x^2+2x+2=0$ (Round off the answer to TWO decimal places) (3)

1.1.2 $x^2+2x-3=5$ (3)

1.1.3 $x^2-2x-15\leq 0$ (3)

1.1.4 $-\sqrt{2x-1}=2-x$ (4)

1.1.5 $7.3^x-3^{x+1}=36$ (3)

1.2 Solve x and y simultaneously:

$2x-1=y$ and $x^2-xy=3x-3$ (5)

1.3 Simplify without using a calculator:

$$\sqrt{5}.\sqrt{125}-\frac{5^x.5^{x+1}}{5^{2x}}$$ (3)

[24]

QUESTION 2

2.1 Consider the arithmetic sequence: $3;-2;-7;-12;...$

2.1.1 Calculate the 21st term of the sequence. (2)

2.1.2 Which term of the sequence is equal to -177? (2)

2.2 The sum of the first terms in an arithmetic series is given by: $S_n=n^2-2n$

Calculate:

2.2.1 the sum of the first 13 terms. (2)

2.2.2 the 13th term. (2)

2.3 Given the quadratic pattern: $x;6;9;y;24;...$

Calculate the sum of x and y. (5)

[13]

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QUESTION 3

3.1 Prove that $a + ar + ar^2 + \ldots$ (to n terms) $= \frac{a(1-r^n)}{1-r}, r \neq 1$ (4)

3.2 Given: $\sum_{k=1}^{\infty} 4(0,2)^{k-1}$

3.2.1 Write down the first THREE terms of the series. (1)

3.2.2 Calculate the sum to infinity of the series. (3)

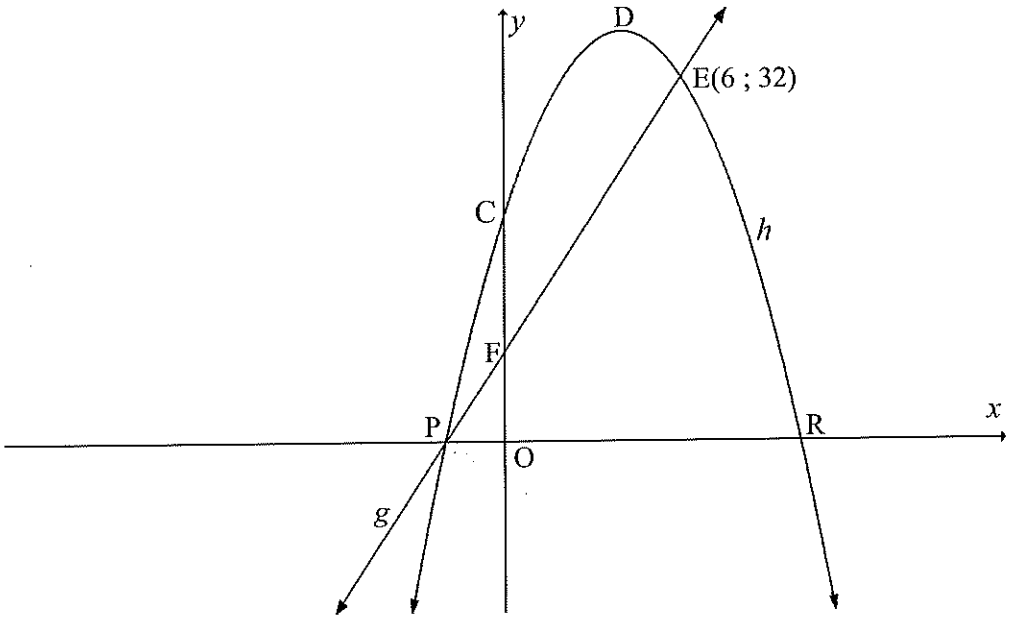
3.2.3 Hence calculate the smallest number of terms of the series whose sum will differ by less than 0,0001 from the sum to infinity of the series. (5)

[13]

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QUESTION 4

In the sketch below, the graphs of $h(x)=ax^2+bx+c$ and $g(x)=4x+8$ are drawn.
P and R are the x-intercepts of h and D is the turning point of h. C and F are the y-intercepts of h and g respectively. The two graphs intersect at P and $E(6;32)$.



4.1 Write down the coordinates of point P. (2)

4.2 If the equation of the axis of symmetry of h is $x=4$, write down the coordinates of point R. (2)

4.3 Determine the equation of h in the form $h(x)=ax^2+bx+c$. (4)

4.4 Calculate the length of FC. (2)

4.5 Determine the range of h. (2)

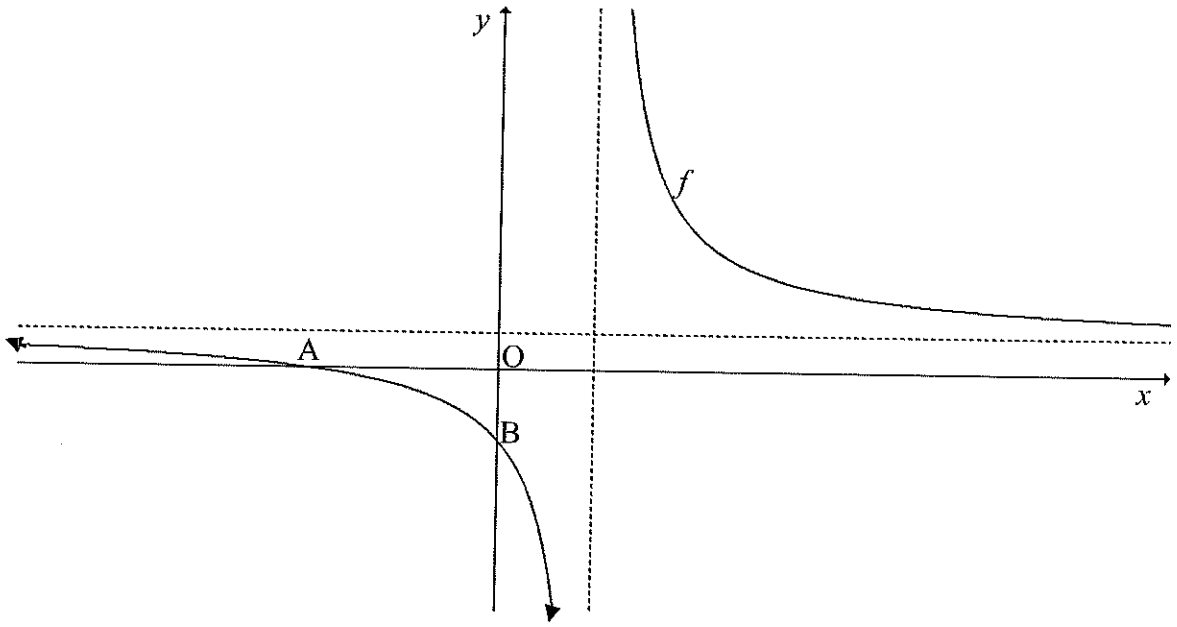
4.6 For which values(s) of x is $h(x).g(x)<0$? (2)

4.7 Determine the value of k if $y=4x+k$ is a tangent to $h(x)=-x^2+8x+20$. (4)
[18]

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QUESTION 5

The sketch below shows the graph of $f(x)=\frac{2+x}{x-1}$. A and B are the x- and the y-intercepts of f.



5.1 Write down the equation of f in the form $f(x)=\frac{a}{x+p}+q$. (3)

5.2 Determine the equations of the asymptotes of f. (2)

5.3 Write down the coordinates of A, the x-intercept of f. (2)

5.4 The graph of $k(x)$ is obtained by shifting the graph of $f(x)$ horizontally such that point A is at the origin, O. Write down the equation of the vertical asymptote of $k(x)$. (1)

[8]

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QUESTION 6

6.1 Given: $f(x)=3^{-x}$

6.1.1 Write down the equation of $f^{-1}(x)$, the inverse of $f(x)$, in the form $f^{-1}(x)=...$ (2)

6.1.2 On the same set of axes, sketch the graphs of $f(x)$ and $f^{-1}(x)$ in your ANSWER BOOK. Clearly indicate the intercepts with the axes and one other point on the graphs and label the TWO graphs. (4)

6.2 A linear function satisfies the following conditions: $p(-3)=10$ and $p'(x)=-2$. Determine the inverse of p in the form $p^{-1}(x)=...$ (4)
[10]

QUESTION 7

7.1 An investment company offers interest on a fixed deposit at the rate of 9,6% compounded monthly. Calculate the effective annual interest rate correct to two decimal places. (3)

7.2 A motor car costing R150 000 depreciated annually at a rate of 8,8% per annum on a reducing-balance method. Calculate how long it took for the car to depreciate to R60 000 under these conditions. (3)

7.3 David is paying off a loan of R250 000 over 10 years by making monthly instalments. He made his first payment exactly one month after the loan is granted. Interest is charged at 10% per annum compounded monthly.

7.3.1 Calculate David's monthly payment. (4)

7.3.2 Calculate the balance outstanding immediately after David paid his 36^{th} instalment. (4)
[14]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=-3x^2$ (4)

8.2 Determine $\frac{dy}{dx}$ if $y=7x^4-5\sqrt{x}-\frac{3}{x}$ (4)

8.3 It is given that $g(x)=ax^3-24x+b$ has a local minimum turning point at $(-2;17)$. Determine the values of a and b. (5)
[13]

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QUESTION 9

Given: $f(x) = -x^3 + 10x^2 - 17x - 28$

9.1 Calculate the coordinates of the intercepts of f with the axes. (5)

9.2 Calculate the coordinates of the turning points of f. (5)

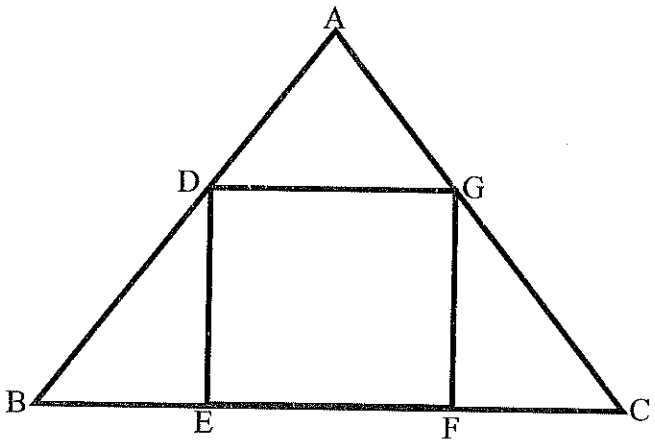
9.3 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (3)

9.4 Determine the values of k for which $-x^3 + 10x^2 - 17x = 25 + k$ will have only one real root. (2)

[15]

QUESTION 10

In the sketch, Δ ABC is an equilateral triangle with sides equal to y units. DEFG is a rectangle. BE = FC = x units.



10.1 Prove that the area of the rectangle is $A = \sqrt{3}xy - 2\sqrt{3}x^2$. (4)

10.2 Determine, in terms of y, the maximum area of the rectangle. (5)

[9]

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QUESTION 11

11.1 $P(A) = 0,45$ and $P(B) = 0,29$. Calculate the P(A *or* B) if:

11.1.1 A and B are mutually exclusive events. (2)

11.1.2 A and B are independent events. (4)

11.2 The probability that it will be sunny tomorrow is $\frac{1}{3}$. If it is sunny, the probability that Pitso plays soccer is $\frac{4}{5}$. If it is not sunny, the probability that Pitso plays soccer is $\frac{2}{5}$. Determine the probability that Pitso does not play soccer. (4)

11.3 How many four character codes can be formed if the first character must be an alphabetical letter and the remaining characters must be digits that may not be repeated? (3)

[13]

TOTAL: 150

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INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$\mathrm{P(A)} = \frac{\mathrm{n(A)}}{\mathrm{n(S)}} \qquad \mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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