education

MPUMALANGA PROVINCE

REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 1

SEPTEMBER 2019

MARKS: 150
TIME: 3 hours

This question paper consist of 9 pages and an information sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **9** questions.
   Answer ALL the questions.
2. Show clearly ALL the calculations, diagrams, graphs, etcetera, which you have used in determining the answers
3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Answers only will not necessarily be awarded full marks.
6. Number the answers correctly according to the numbering system used in this question paper.
7. Diagrams and graphs are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the Question Paper.
9. Write neatly and legibly.

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QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 2x = 0$ (3)

1.1.2 $2x - \frac{8}{x+1} = 3$ (correct to 2 decimal places) (5)

1.1.3 $2x^2 + x - 3 > 0$ (3)

1.1.4 $x^{\frac{2}{3}} - x^{\frac{1}{3}} = 6$ (3)

1.2 Given: $4^{x+2} \cdot 8^{y+1} = 2^{1-x}$ and

$x^2 + y^2 + xy = 7$

1.2.1 Show that $y = -x - 2$ (3)

1.2.2 Hence, solve for x and y simultaneously. (5)

1.3 If the roots of $mx^2 + 5x + 4 = 0$ are non-real, calculate the lowest integral value of m. (4)

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QUESTION 2

2.1 A quadratic number pattern and a constant value is combined to form the sequence:

8 ; 1 ; 18 ; 1 ; 30 ; 1 ; 44 ; 1 ; ....

2.1.1 Write down the next TWO terms of the sequence. (1)

2.1.2 Calculate the n^{th} term of the quadratic sequence. (4)

2.1.3 Which term of the given sequence is 368? (4)

2.2 The sum of the second and third terms of a geometric series is 280 and the sum of the fifth and sixth terms is 4375.

Determine the common ratio. (5)

2.3 The first term of an arithmetic series is increased by 3, but the common difference remains the same. By how much will the sum of the first twelve terms increase? (4)

2.4 A converging geometric series is given by

$1 + 4m + 16m^2 + ...$

2.4.1 Determine the values of m for which the given series will converge. (3)

2.4.2 Calculate the value of m if $1 + 4m + 16m^2 + ... = \frac{2}{3}$ (3)

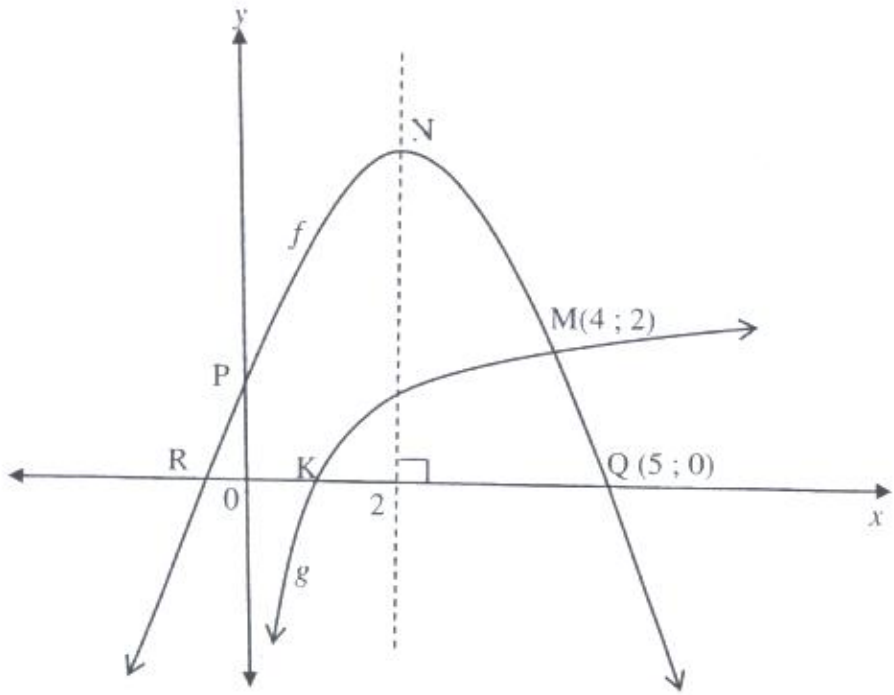
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QUESTION 3

In the diagram below, the graph of $f(x) = ax^2 + bx + c$ and $g(x) = \log_m x$ intersect at M(4 ; 2)

The axis of symmetry of f is $x = 2$ .N is the turning point of the parabola. R and Q (5 ; 0) are the x- intercepts of f and P is the intercept of f. K is the x- intercept of g.



3.1 Determine the value of m. (2)

3.2 Write down the equation of $g^{-1}(x)$, the inverse of g, in the form $y = ...$ (2)

3.3 Determine the equation of the parabola in the form $f(x) = ax^2 + bx + c$. (5)

3.4 Determine the value of $f(0) - g^{-1}(0)$. (1)

3.5 Write down the co-ordinates of the turning point of $f(x+3)$ (4)

3.6 Determine the values of x for which:

3.6.1 $f'(x) \cdot g(x) \geq 0$ (2)

3.6.2 $f(x) - \frac{13}{5} = g(x)$ (2)

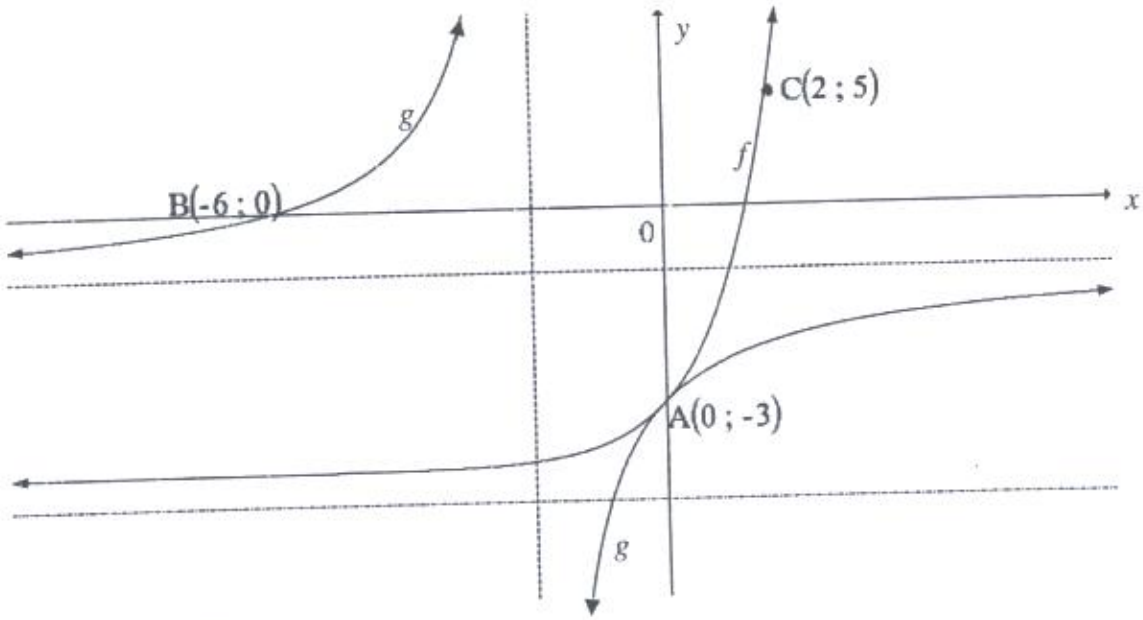
[18]

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QUESTION 4

In the diagram, the graphs of $f(x) = m^x + k$ and $g(x) = \frac{-4}{x+2} - 1$ are drawn.

The two graphs intersect at $A(0; -3)$. The point $C(2; 5)$ lies on f and $B(-6;0)$ is the x- intercept of g.



4.1 Determine:

4.1.1 the equation of f (4)

4.1.2 the equation of h, the axis of symmetry of g with a negative gradient (3)

4.2 Describe the transformation that g has to undergo to form the graph of

$$p(x) = \frac{-4}{x+4} + 4$$ (2)

4.3 For which value of x is:

$h(x) \leq g(x)$? (5)

[14]

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QUESTION 5

5.1 John wishes to save money for his child's education. He will deposit R500 into a savings account on his child day of birth. Thereafter he deposits R500 at the end of each month for the next 18 years.
Determine the final value of this investment, if the rate of interest is 9% p.a. compounded monthly? (4)

5.2 John invests R600 into an account earning 8% per annum, compounded quarterly. His friend, Peter, also invests R600 and earns interest compounded semi-annually (every 6 months). John and Peter's investments are the same at the end of 12 months. Calculate the interest rate that Peter receive. (4)

5.3 The Mahlangu family is eager to possess their own home. They can afford a monthly payment of R4000. They want to buy a house for R650 000 and the bank offers them a home loan at 14,3% p.a. compounded monthly, over a period of 20 years.
Will they be able to afford the buy the house? (4)

5.4 Mr Brown receives an amount of R1 000 000 at retirement which he invests at a rate of 9,5% per annum, compounded monthly. His monthly expenditure is R25 000 per month to keep up his normal way of living.
For how many months will he be able to live from his investment? (4)

[16]

QUESTION 6

6.1 Given: $f(x) = -2x^2 + 2$

Determine $f'(x)$ from FIRST PRINCIPLES. (5)

6.2 Determine:

6.2.1 $\frac{dy}{dx}$, if $y = (x-2)^2$ (3)

6.2.2 $f'(x)$ if $f(x) = \frac{2x^2 + \sqrt{x^5}}{6x}$ (4)

[12]

QUESTION 7

7.1 The cubic graph , f, has the following properties.

$f'(x) > 0$ when $x < 1$ and $x > 5$

$f'(x) < 0$ when $1 < x < 5$

$f'(1) = 0$ and $f'(5) = 0$

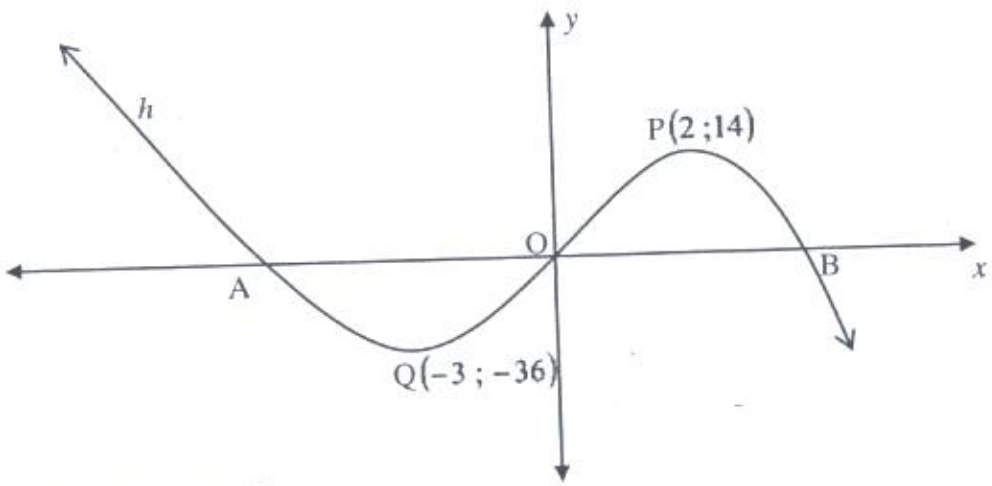
$f(3) = 0$ and $f(0) = -12$

Sketch the graph of f, showing all the critical points. (4)

7.2 In the diagram, the graph of $h(x) = -x^3 - 2x^2 + cx$ is drawn.

Graph h cuts the x- axis at A and B and the origin.

$P(2;14)$ and $Q(-3;-36)$ are the turning points of h.



7.2.1 Show that $c = 15$. (2)

7.2.2 Determine the length of AB (3)

7.2.3 For which value(s) of n will the equation $h(x) = -x^3 - 2x^2 + 15x$ have only one negative root? (2)

7.2.4 Determine the interval on which h will be concave up. (3)

7.2.5 Determine the x coordinate where the tangent, parallel to the AP will touch the graph of h. (5)

[19]

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QUESTION 8

The depth, D, of water in a kettle t minutes after it starts to boil, is given by $D(t) = 86 - \frac{1}{8}t - \frac{1}{4}t^3$, where D is measured in millimetres.

8.1 What is the depth of the water in the kettle just before it starts to boil? (1)

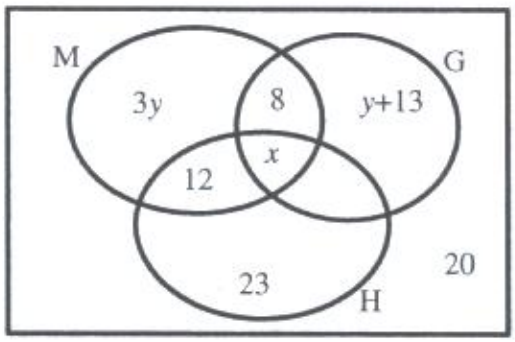
8.2 As the water boils, the level in the kettle drops. Find the rate at which the water level is decreasing when $t = 2$ minutes. (3)

8.3 How many minutes after the kettle starts boiling will the water level be dropping at a rate of 12⅛ ml/minute? (2)

[6]

QUESTION 9

9.1 In St Peter's School there are 150 learners in grade 12. The Venn-diagram shows the learners that take Geography (G), Mathematics (M) and History (H). The total number of learners taking History is 49 and those taking Mathematics is 74.



9.1.1 Determine the value of x and y. (4)

9.1.2 Calculate the probability that if a grade 12 learner is randomly selected, the learner takes Mathematics and History, but not Geography. (2)

9.1.3 Are H and G independent events? Give a reason for your answer. (4)

9.2 Lucky, Simon and 5 friends sits in a row of seven seats at the theatre.

9.2.1 In how many different ways can the seven people sit if any person occupies any of the seven seats? (2)

9.2.2 In how many different ways can the seven people sit if Lucky and Simon do not sit next to each other? (3)

[15]

TOTAL: 150