GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION
GRADE 12

2023

TECHNICAL MATHEMATICS

(PAPER 1)

TIME: **3 hours**

MARKS: **150**

10 pages, a 2-page information sheet and 1 answer sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Answer QUESTION 4.3 on the ANSWER SHEET provided. Write your NAME and SURNAME in the spaces provided on the ANSWER SHEET and hand in the ANSWER SHEET with your ANSWER BOOK.
4. Number the answers correctly according to the numbering system used in this question paper.
5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

QUESTION 1: ALGEBRA

1.1 Solve for x:

1.1.1 $x^2 + 25 = 0$ ($x \in$ Complex numbers) (3)

1.1.2 $-3x^2 = -2x - 2$ (Correct to TWO decimal places) (4)

1.1.3 $x^2 - 2x - 15 \leq 0$ (3)

1.2 Solve for x and y if:

$2x - 1 = y$ and $x^2 - xy = 3x - 3$ ... (5)

1.3 $N = N_0e^{kt}$ where k > 0 describes the population N of a colony of bacteria at time t with N_0 as the initial population and a growth rate of k.

1.3.1 Make the initial population N_0 the subject of the formula. (2)

1.3.2 At the start of an experiment, there are 6 000 bacteria, and one hour later, the population has increased to 6 400. How long will it take for the population reach 100 000 if k = 0,065? Write down only the equation which needs to be solved. DO NOT perform the calculations. (2)

1.4 Given the binary numbers:

$S = 100111_2$ and $T = 10000_2$

1.4.1 Determine $S + T$ in binary form. (2)

1.4.2 Write the answer of QUESTION 1.4.1 in decimal form. (1)

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QUESTION 2: NATURE OF ROOTS

2.1 The solution of a quadratic equation is $x = \dfrac{-2 \pm \sqrt{10 - 20k}}{10}$ where $k \in \mathbb{Q}$.

Determine the value(s) of k so that:

2.1.1 The equation has equal roots. (2)

2.1.2 The equation has non-real roots. (2)

2.2 Determine the nature of the roots value of $x^2 - 5x - 7 = 0$. (4)

[8]

QUESTION 3: LOG LAWS, EXPONENTS, SURDS AND COMPLEX NUMBERS

3.1 Simplify the following **without the use of a calculator**:

3.1.1 $\left(729x^{-6}\right)^{-\frac{1}{6}}$ (Leave the answer with positive exponents.) (2)

3.1.2 $\dfrac{\sqrt{5}}{1-\sqrt{5}}$ (3)

3.2 Simplify the following **without the use of a calculator:**

3.2.1 $\log\ 1 \times \log\ 10 \times \log\ 100$ (2)

3.2.2 $\log_3(27 \times 81)$ (3)

3.3 Solve for x: $\log(x+2) + \log(x-1) = 1$. (5)

3.4 Given complex numbers: $z = 2w - 7i$ where $w = \frac{1}{2} + 3i$.

3.4.1 Determine z in die form $a + bi$. (2)

3.4.2 Determine the modules. (1)

3.4.3 Determine the argument. (1)

3.4.4 Write z in polar form z = r *cis* θ (where θ is in degrees). (1)

3.5 Solve for x and y if $x + i = (1 - yi)(2 + 3i)$. (4)

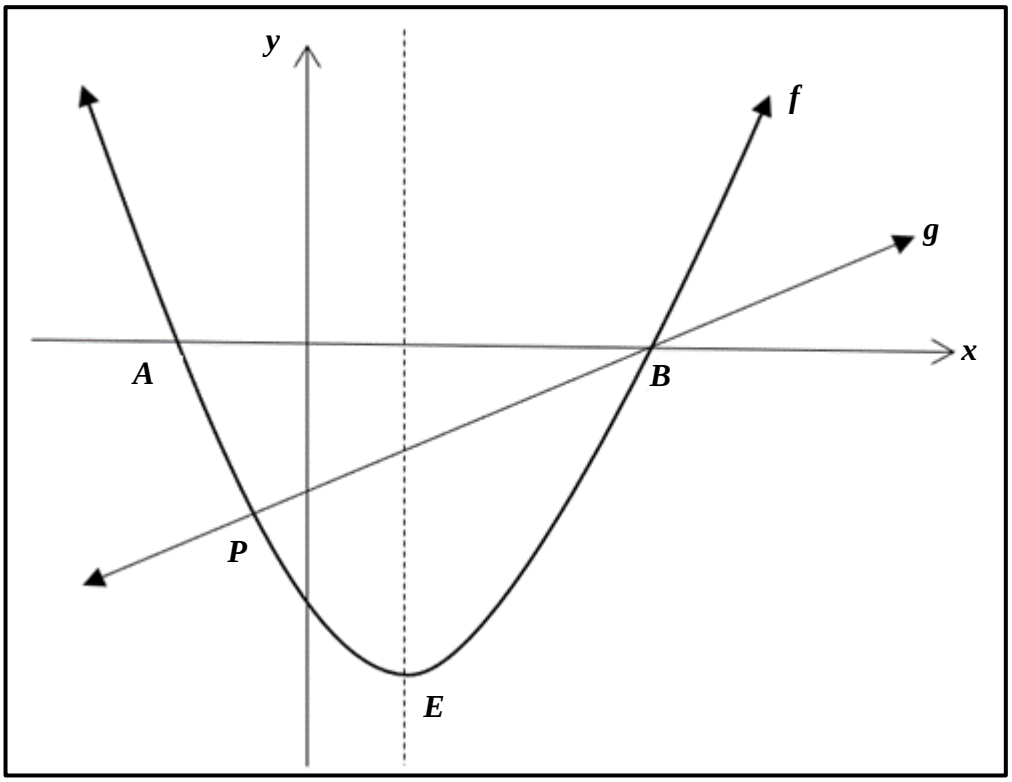
[24]

QUESTION 4: FUNCTIONS

4.1 Sketched below are the graphs of functions f and g defined by:

$f(x) = x^2 - 6x - 16$ and $g(x) = x - 8$

- E is the turning point of f.
- Points P and B are the intercepts of f and g
- A and B are points on the x-axis.



Determine:

4.1.1 The coordinates of A and B (4)

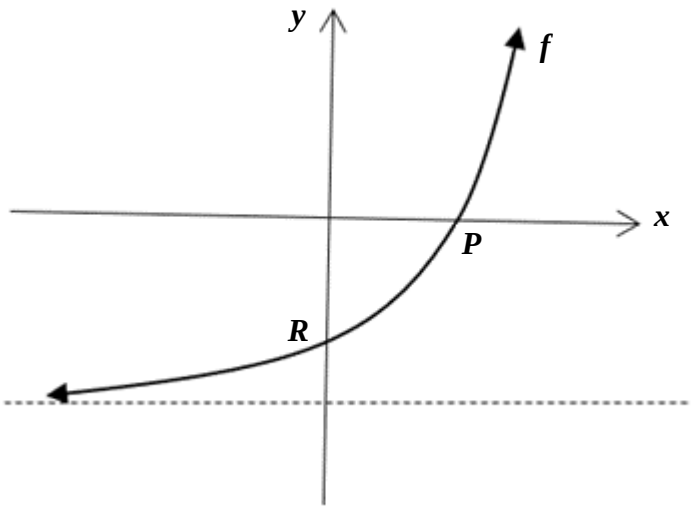
4.1.2 The coordinates of E, the turning point of f (4)

4.1.3 The coordinates of P (6)

4.1.4 The length of AB (1)

4.1.5 For what value(s) of x is $f(x) < g(x)$ (2)

4.2 Sketched below is the graph of $f(x) = 2^x - 4$ with P and R the intercepts with the axes.



4.2.1 Write down the equation of the asymptote for this function. (1)

4.2.2 Determine the coordinates of P. (2)

4.2.3 Determine the coordinates of R. (2)

4.2.4 Write down the range of $f(x)$. (2)

4.3 Given: $f(x) = \sqrt{9 - x^2}$ and $g(x) = \frac{-4}{x} - 3$.

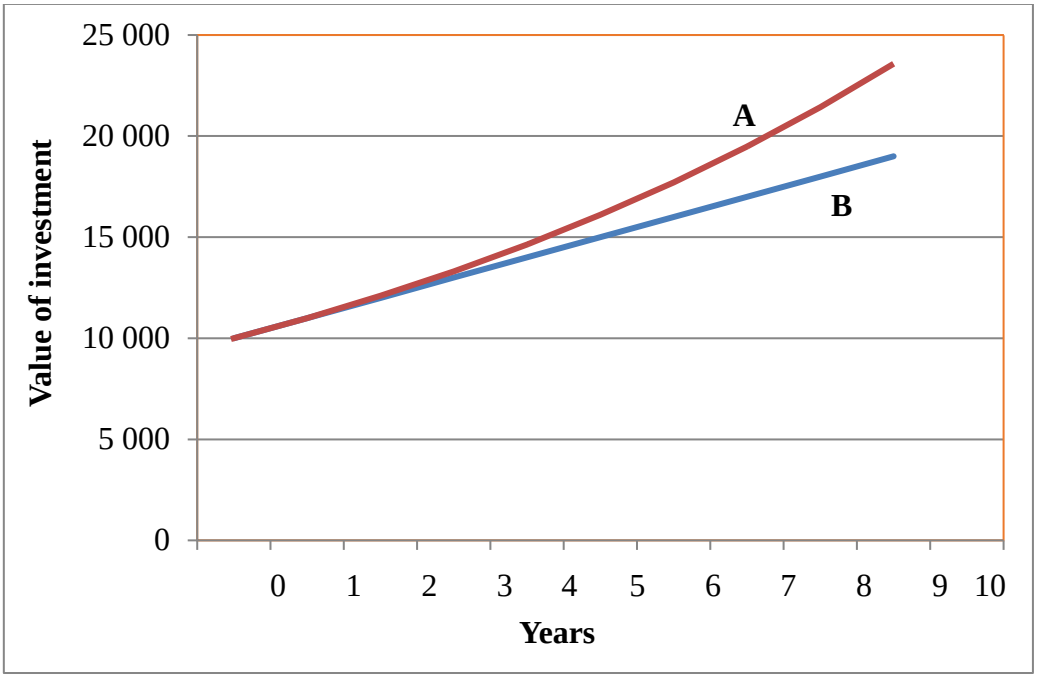
Sketch the graphs of f and g on the same set of axes on the ANSWER SHEET provided. Clearly show the intercepts with the axes and any asymptotes. (6)

[30]

QUESTION 5: FINANCE

5.1 Choose the graph (A or B) that best fits the descriptions below. Write down only the question number and the correct letter.

GROWTH OF INVESTMENTS



5.1.1 The growth of an investment of R10 000 at simple interest of 12% per annum. Motivate your choice. (2)

5.1.2 The growth of an investment of R10 000 at an interest rate of 12% p.a. compounded annually. Motivate your choice. (2)

5.2 Two businessmen, Joseph and Xolani invested R12 000 each in their respective banks on the same day. Joseph invested his money for 10 years at a rate of 7,5% **simple interest** per year while Xolani invested his money for the same period at a rate of 6,96% interest per year **compounded monthly**.

Answer the following questions.

5.2.1 After approximately how many years will Joseph have R18 000 in the bank? (3)

5.2.2 On 1 January 2018, Xolani invests an amount of R12 000 in an account that pays 6,96% p.a. interest compounded monthly. At the beginning of 2020, he withdraws R3 800 from the account. On 1 January 2022 the interest rate of the account changes to 13,2% p.a. compounded quarterly. He pays another R4 200 into the account on 1 January 2022 and plans to pay R6 800 into the account again on 1 January 2025.
Calculate the balance in the account on 31 December 2021, just before the interest rate changes. Show ALL calculations. (6)

5.3 Calculate the effective interest rate if interest is 9,8% p.a. compounded monthly. (3)

5.4 A tractor which was bought for R120 000 depreciates over a period of 12 years using the reducing balance method. Calculate the annual depreciation rate if the value after the term is R11 090,41. (The rate was unchanged for the 12 years.) (4)

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QUESTION 6: CALCULUS

6.1 Determine $f'(x)$ from FIRST PRINCIPLES if $f(x) = 2x + 3$. (5)

6.2 Determine:

6.2.1 $\frac{dy}{dx}$ if $y = -x^{-5} + 3x^4$ (2)

6.2.2 $f'(x)$ if $f(x) = \frac{3}{x^4} - \frac{x}{\sqrt{x}}$ (4)

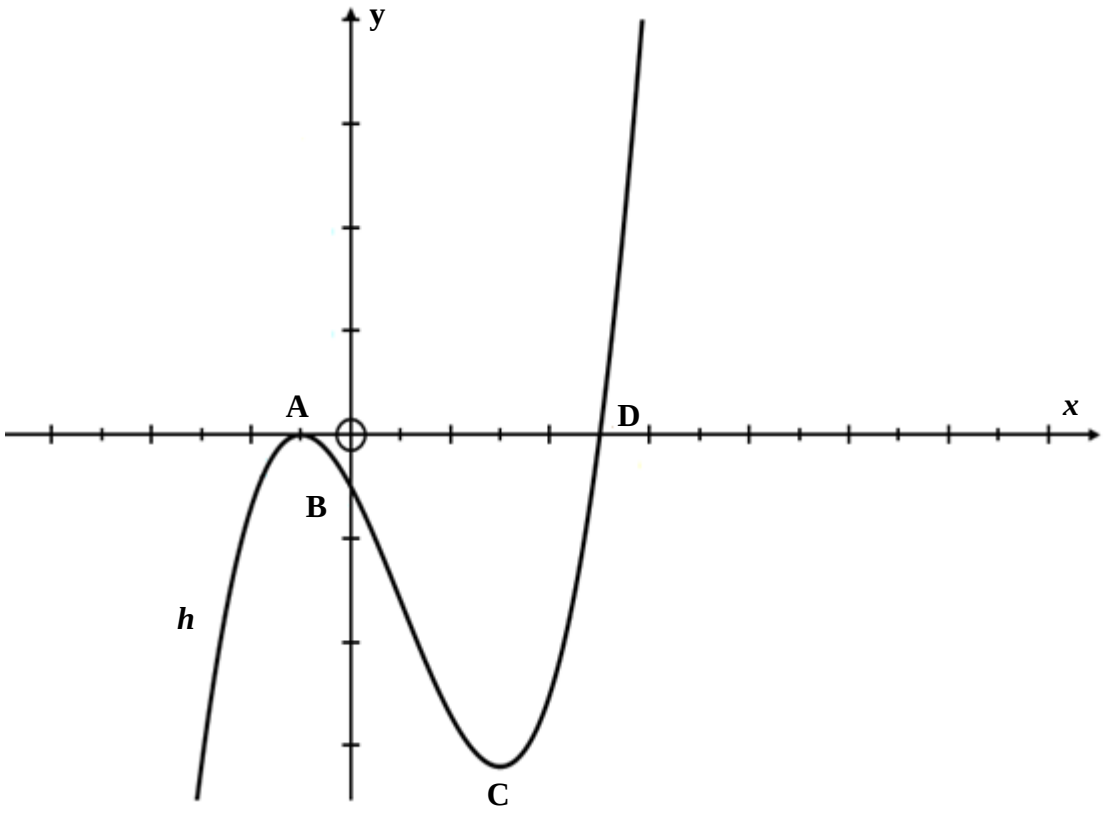
6.2.3 $D_x\left[\frac{x^2 + x - 6}{x + 3}\right]$ (3)

6.3 Determine the average gradient of the function defined by $h(x) = -2x^2 + 2$ between the points where $x = 0$ and $x = 2$. (3)

[17]

QUESTION 7: CUBIC FUNCTIONS

The graph below represents the function defined by $h(x) = x^3 - 3x^2 - 9x - 5$.
A and C are the turning points of h.
A and D cut the x-axis.
The graph cuts the y-axis at point B.



7.1 Write down the coordinates of point B. (1)

7.2 Show that $x + 1$ is a factor of h. (2)

7.3 Hence, determine the coordinates of point D. (3)

7.4 Determine the coordinates of turning point C. (5)

7.5 Write down the value(s) of x for which $h(x) > 0$. (2)

[13]

QUESTION 8: CALCULUS

A printer produces bumper stickers for an election campaign. The cost to produce one sticker is $0,15 - 0,000002x$ rand, if x stickers are ordered (with $x < 10\ 000$) and the total cost to deliver the order is $0,095x - 0,0000005x^2$ rand.

8.1 Express the revenue in terms of x.
Use the fact that revenue = price per item × number of items sold to express the revenue R(x) from x stickers as a product of two functions of x. (2)

8.2 Determine the revenue when 5 000 stickers were produced. (2)

8.3 Determine the maximum revenue. (3)
[7]

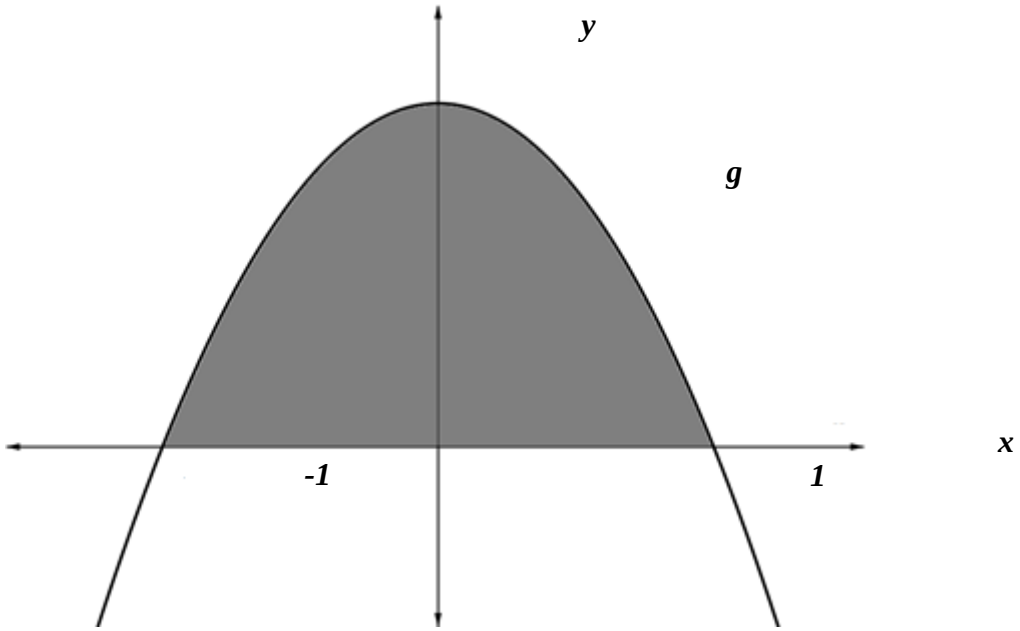
QUESTION 9: INTEGRATION

9.1 Determine the following integrals:

$$\int\left(x^{-2}+\frac{1}{x}\right)\,dx$$

(3)

9.2 The sketch below shows the shaded area bounded by function g defined by $g(x) = -3x^2 + 3$.



Calculate the area of the shaded part which is included between the graph $g(x) = -3x^2 + 3$ and the x-axis between $x = -1$ and $x = 1$. (6)
[9]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0,\ a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1+i)^n \qquad A = P(1-i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int k\,x^n dx = \frac{k\,x^{n+1}}{n+1} + C \quad , \ n, k \in \mathbb{R} \text{ with } n \neq -1 \text{ and } k \neq 0$$

$$\int \frac{k}{x} dx = k \ln x + C \quad , \ x > 0 \text{ and } k \in \mathbb{R};\ k \neq 0$$

$$\int k\,a^{nx} dx = \frac{k\,a^{nx}}{n \ln a} + C \quad , \ a > 0;\ a \neq 1 \text{ and } k, a \in \mathbb{R};\ k \neq 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_2 + x_1}{2}\ ;\ \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \tan\theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$$\text{area of } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$\pi\, rad = 180°$

Angular velocity $= \omega = 2\,\pi\, n$ where n = rotation frequency

Angular velocity $= \omega = 360°\, n$ where n = rotation frequency

Circumferential velocity $= v = \pi\, D\, n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega\, r$ where ω = angular velocity and r = radius

Arc length $= s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{r\,s}{2}$ where r = radius, s = arc length

Area of a sector $= \dfrac{r^2\,\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4d\,h + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a\left(m_1 + m_2 + m_3 + ... + m_n\right)$ where a = number of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$

$O_n = n^{th}$ ordinate and n = number of ordinates

OR

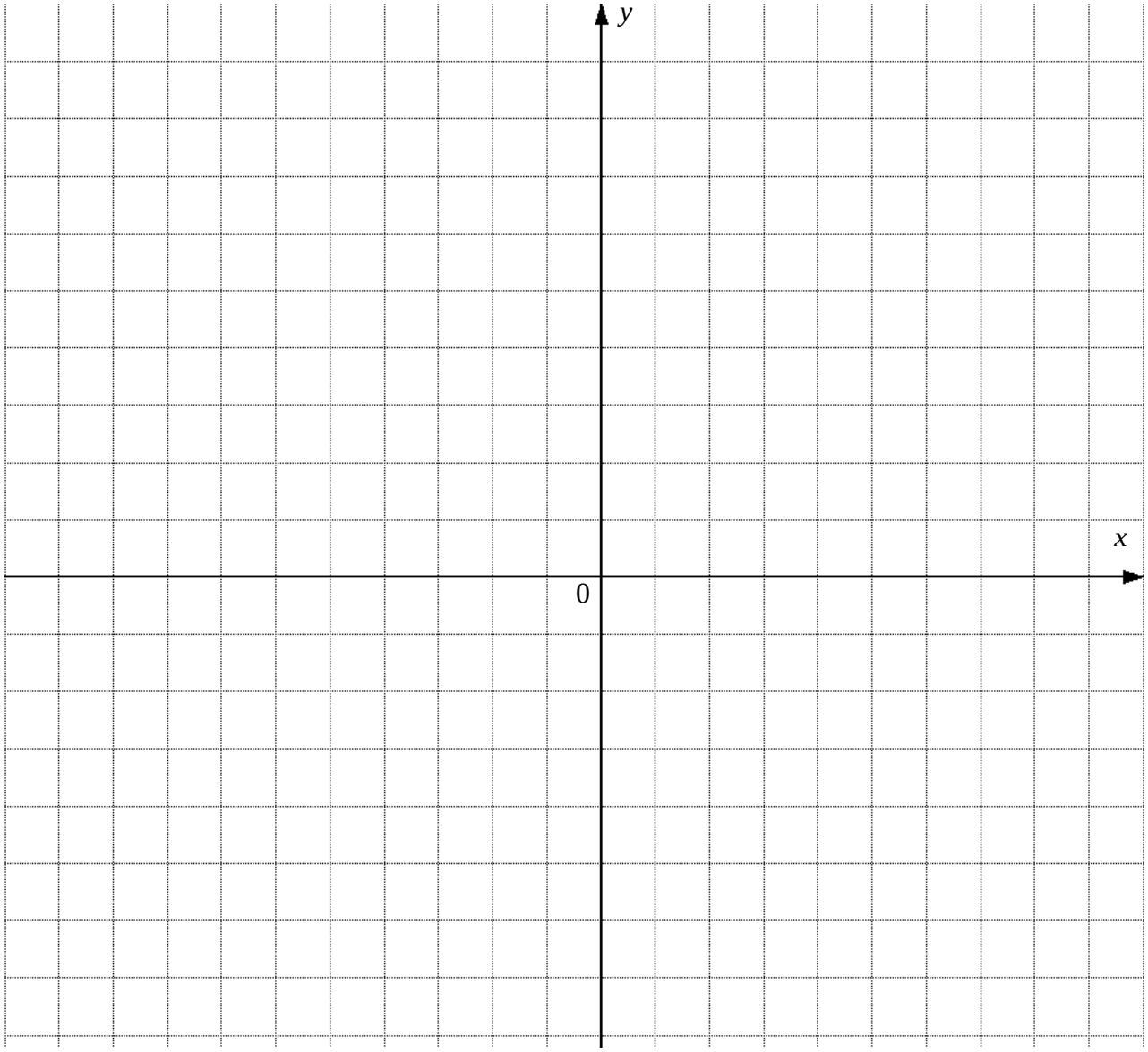
$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = number of equal parts, $o_n = n^{th}$ ordinate and n = number of ordinates

ANSWER SHEET

NAME:	

SURNAME:	

QUESTION 4.3



(6)